

SUSY flavor structure of generic 5D supergravity models

Hiroyuki Abe,^{1*} Hajime Otsuka,^{1†} Yutaka Sakamura,^{2‡}
and Yusuke Yamada^{1§}

¹*Department of Physics, Waseda University,
Tokyo 169-8555, Japan*

²*KEK Theory Center, Institute of Particle and Nuclear Studies, KEK,
Tsukuba, Ibaraki 305-0801, Japan*

*Department of Particles and Nuclear Physics,
The Graduate University for Advanced Studies (Sokendai),
Tsukuba, Ibaraki 305-0801, Japan*

Abstract

We perform a comprehensive and systematic analysis of the SUSY flavor structure of generic 5D supergravity models on S^1/Z_2 with multiple Z_2 -odd vector multiplets that generate multiple moduli. The SUSY flavor problem can be avoided due to contact terms in the 4D effective Kähler potential peculiar to the multi-moduli case. A detailed phenomenological analysis is provided based on an illustrative model.

*E-mail address: abe@waseda.jp

†E-mail address: hajime.13.gologo@akane.waseda.jp

‡E-mail address: sakamura@post.kek.jp

§E-mail address: yuusuke-yamada@asagi.waseda.jp

1 Introduction

The standard model (SM) of elementary particles is a successful theory without any contradiction to the observations up to now. However, it contains many free parameters, most of which come from Yukawa couplings. The eigenvalues and eigenvectors of the Yukawa coupling matrices determine the mass ratios and mixings between generations, respectively, and their observed values are quite hierarchical. Models beyond the SM should explain such hierarchical structures of quarks and leptons.

Extra dimensions provide a simple way to realize the hierarchical flavor structures, *i.e.*, a wave function localization of matter fields in extra dimensions [1]. Actually, the most promising candidate for the unified theory of the SM and the gravity, *i.e.*, the superstring theory, predicts the existence of extra dimensions. They can also explain other problems of the SM, such as the gauge hierarchy problem [2], a candidate for dark matter [3], and so on.

Supersymmetry (SUSY) is an interesting extension of the SM. It softens the divergences in quantum field theories and then protects the electroweak scale against large radiative corrections. The three SM gauge couplings are unified in the minimal supersymmetric standard model (MSSM) at $M_{\text{GUT}} \equiv 2 \times 10^{16}$ GeV, which suggests the grand unified theory. It also has a candidate for dark matter if the R-parity forbids decays of the lightest SUSY particle. Besides, the existence of SUSY is predicted by the superstring theory.

From the above reasons, we consider an extension of the SM by introducing SUSY and extra dimensions. The minimal setup for such an extension is a five-dimensional (5D) supergravity (SUGRA) compactified on an orbifold S^1/Z_2 . The chiral structure of the SM can be realized by the orbifold Z_2 projection, which preserves $N = 1$ SUSY in a four-dimensional (4D) sense. The local SUSY (*i.e.*, SUGRA) is required in order to avoid the existence of a massless Goldstino which contradicts to many observations, and to discuss the moduli stabilization.

The off-shell formulation of 5D SUGRA [4, 5] provides the most general way to construct 5D SUGRA models. There is, moreover, a systematic way to obtain 4D effective theories of such models, which we call the off-shell dimensional reduction [6] based on the $N = 1$ superfield description of 5D SUGRA [7, 8]. This method can be applied to general 5D SUGRA models. For example, we analyzed some class of models by this method [9] that include SUSY extension of the Randall-Sundrum model [10] and the 5D heterotic M theory [11] as special limits of parameters.

The wave function localization of matter fields (hypermultiplets) is realized by controlling bulk mass parameters for the matter fields in 5D models. In 5D SUGRA on S^1/Z_2 , the bulk mass parameters are obtained as $U(1)$ charges (times 5D Planck mass) under Z_2 -odd vector multiplets including the graviphoton multiplet. One of the minimal models with the realistic flavor structure was constructed with a single Z_2 -odd vector multiplet. However, it suffers from the SUSY flavor problem as well as tachyonic squarks and/or sleptons [12]. In our previous paper [13], we constructed models with two Z_2 -odd vector multiplets, which induce a modulus chiral multiplet other than the radion multiplet in the 4D effective theory, and showed that there is an important contribution of the multiple moduli multiplets to the effective Kähler potential that may solve problems mentioned above. In this paper, we extend the previous work to more generic set-up, which has an

arbitrary number of Z_2 -odd vector multiplets (*i.e.*, moduli multiplets) and a nontrivial warping along the extra dimension. We perform a comprehensive and systematic analysis to understand the SUSY flavor structure of such generic 5D SUGRA models, and provide a phenomenological analysis based on an illustrative model.

This paper is organized as follows. In Sec. 2, we set up our model with a brief review of the off-shell formulation of 5D SUGRA. In Sec. 3, we derive 4D effective theory of our 5D model and study its properties. In Sec. 4, we perform a phenomenological analysis based on an illustrative model. Sec. 5 is devoted to a summary. In Appendix A, some details of the derivation of the effective action are shown. In Appendix B, we provide a comment on some peculiar structure of the 4D effective theory to 5D SUGRA. In Appendix C, explicit expressions of some quantities in the model in Sec. 4 are collected.

2 Set-up and brief review of 5D SUGRA

2.1 Brief picture of set-up

The set-up we consider in this article is as follows.

- The fifth dimension is compactified on the orbifold S^1/Z_2 , and the background 5D metric is the warped metric. In contrast to the original warped model [2], the warp factor is supposed to be $\mathcal{O}(10^2)$ and explains the small hierarchy between the Planck scale $M_{\text{Pl}} = 2.4 \times 10^{18}$ GeV and the GUT scale $M_{\text{GUT}} = 2 \times 10^{16}$ GeV.
- The compactification scale is around M_{GUT} , and below this scale, 4D effective theory becomes MSSM.
- All the standard model fields are identified with zero-modes of the 5D bulk fields.
- The hierarchical flavor structure of the SM fermions is realized by the quasi-localization of the wave functions for the zero-modes [1].
- SUSY is broken at some scale below M_{GUT} , and the main source of SUSY breaking is the F -term of a single chiral superfield X in 4D effective theory.

When we study a model with extra dimensions, it is indispensable to stabilize the size of the compactified internal space to a finite value. Especially in SUSY models, such stabilization mechanisms affect the sfermion mass spectrum in the MSSM sector. In order to take into account the stabilization of the extra dimension, we have to work in the context of SUGRA.

Although the above type of set-up has been studied in many papers, most of them do not consider the full SUGRA effects or only consider a *limited* case from the viewpoint of SUGRA. In our previous work [13], we pointed out a possibility to solve the SUSY flavor problem thanks to some peculiar terms in the Kähler potential of 4D effective theory, in a case that the theory has two moduli multiplets (we will provide the definition of the moduli multiplet in the next subsection).

In this article, we extend our previous work to the case of an arbitrary number of the moduli multiplets and a non-trivial warp factor, and discuss some phenomenological

5D superconformal multiplet	$N = 1$ decomposition	Z_2 -parity
Weyl multiplet (gravity)	$\mathbf{E}_W = (E_W, L_E^\alpha, V_E)$	$(+, -, +)$
Vector multiplet (moduli)	$\mathbf{V}^{I'} = (V^{I'}, \Sigma^{I'})$	$(-, +)$
Vector multiplet (gauge)	$\mathbf{V}^{I''} = (V^{I''}, \Sigma^{I''})$	$(+, -)$
Hypermultiplet (compensator)	$\mathbf{H}^{a=1} = (\Phi^1, \Phi^2)$	$(-, +)$
Hypermultiplet (matter)	$\mathbf{H}^{a \geq 2} = (\Phi^{2a-1}, \Phi^{2a})$	$(-, +)$

Table 1: Relevant 5D superconformal multiplets. Each multiplet is decomposed into $N = 1$ multiplets.

aspects. Before specifying the model, we start with *general* 5D SUGRA, derive 4D effective theory in a systematic way, and analyze the flavor structure of the soft SUSY breaking terms in the effective theory. After that, we will construct an illustrative model in Sec 4 to realize the above set-up. For our purpose, the superconformal gravity formulation, which is an off-shell description of 5D SUGRA, is useful. So we briefly review this formulation and explain the structure of 5D SUGRA in the rest of this section.

2.2 Superconformal multiplets in 5D SUGRA

The background 5D metric with 4D Poincaré invariance is parametrized as

$$ds^2 = e^{2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad (2.1)$$

where $\mu, \nu = 0, 1, 2, 3$, $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, and $e^{\sigma(y)}$ is a warp factor, which is a function of only y and determined by the dynamics. We take the fundamental region of the orbifold as $0 \leq y \leq L$. Since we choose the coordinate y such that $\langle e_y^4 \rangle = 1$, the constant L denotes the size of the extra dimension.

Our formalism is based on the superconformal formulation developed in Ref. [4, 5]. 5D superconformal multiplets relevant to our study are summarized in Table 1. Each multiplet can be decomposed into $N = 1$ multiplets [5]. The signs in the last column of Table 1 denote the orbifold Z_2 parities of the $N = 1$ multiplets in the next column. We assume that each $N = 1$ multiplet has the same Z_2 parity at both boundaries $y = 0, L$, for simplicity.¹

Weyl multiplet \mathbf{E}_W This corresponds to the gravitational multiplet, and is decomposed into the $N = 1$ Weyl multiplet E_W , a complex general multiplet L_E^α (α : spinor index), and a real general multiplet V_E . Among them, E_W and V_E are Z_2 -even, and includes the 4D parts of the vierbein e_μ^ν and the extra-dimensional component e_y^4 , respectively. The Z_2 -odd multiplet L_E^α includes the “off-diagonal” parts e_μ^α and e_y^μ .

¹ In general, vector and hypermultiplets can have different Z_2 parities at different boundaries, but only Z_2 -even fields at both boundaries have zero-modes which are relevant to 4D effective theory. All the other fields are decoupled and should be integrated out. Thus the treatment of multiplets with different Z_2 parities is the same as that of multiplets which are Z_2 -odd at both boundaries.

The latter is irrelevant to the following discussion, and is neglected. When the loop corrections are taken into account, however, the contribution from L_E^α has to be included.

Vector multiplet V^I This is decomposed into $N = 1$ vector and chiral multiplets V^I and Σ^I , which have opposite Z_2 -parities. The vector multiplets are divided into two classes according to their Z_2 parities. One is a class of the gauge multiplets, which are denoted as $V^{I''}$. In this class, $V^{I''}$ are Z_2 -even and have zero-modes that are identified with the gauge multiplets in 4D effective theory. In the other classes ($V^{I'}$), the 4D vector components have no zero-modes. Instead, the chiral multiplets $\Sigma^{I'}$ have zero-modes. They include the scalar fields and their potential is flat at the classical level. Thus we refer to $\Sigma^{I'}$ (or $V^{I'}$) as the moduli multiplets in this article.²

At least one vector multiplet belongs to the latter category. In the pure SUGRA, the vector component of such a multiplet is identified with the graviphoton.³ All the other components are auxiliary fields that are eliminated by the superconformal gauge-fixing. Thus, when there are n_V moduli multiplets in the off-shell action, only $(n_V - 1)$ degrees of freedom are physical. (See Sec. 2.4.)

Hypermultiplet H^a This is decomposed into two chiral multiplets Φ^{2a-1} and Φ^{2a} , which have opposite Z_2 -parities. We can always choose their Z_2 -parities as listed in Table 1 by using $SU(2)_U$, which is an automorphism of the superconformal algebra. The hypermultiplets are also divided into two classes. One is the compensator multiplets $a = 1, 2, \dots, n_C$ and the other is the physical matter multiplets $a = n_C + 1, \dots, n_C + n_H$. The former is an auxiliary degree of freedom and eliminated by the superconformal gauge-fixing. In contrast to 4D SUGRA, both types of hypermultiplets have the same quantum numbers of the superconformal symmetries in 5D SUGRA. They are discriminated only by signs of their kinetic terms in the action. Thus, in principle, it is possible to introduce an arbitrary number of the compensator multiplets in the theory. In this article, we consider the single compensator case ($n_C = 1$) for simplicity.⁴

2.3 $N = 1$ description of 5D action

For our purpose, it is convenient to describe the 5D action in terms of the $N = 1$ multiplets [7, 8]. This corresponds to the extension of the result in Ref. [14] to the local SUSY case. We can see that V_E has no kinetic term in this description.⁵ After integrating it out,

² These moduli fields are actually identified with the shape moduli of the compactified space for a 5D effective theory of the heterotic M-theory compactified on the Calabi-Yau manifold [11], for example.

³ In this article, the terminology “graviphoton” represents the vector field in the gravitational multiplet of the *on-shell* formulation. It should be distinguished from the off-diagonal components of the 5D metric, which are included in L_E^α in the current formulation.

⁴ The number of the compensator multiplets determines the target manifold of the hyperscalars. For instance, it is $USp(2, 2n_H)/USp(2) \times USp(2n_H)$ for $n_C = 1$, and $SU(2, n_H)/SU(2) \times U(n_H)$ for $n_C = 2$.

⁵ This does not mean that e_y ⁴ is an auxiliary field. It is also contained in Σ^I , which have their own kinetic terms.

the 5D Lagrangian is expressed as [15]

$$\begin{aligned} \mathcal{L} = & -3e^{2\sigma} \int d^4\theta \mathcal{N}^{1/3}(\mathcal{V}) \left\{ d_{\hat{a}}^{\hat{b}} \bar{\Phi}_{\hat{b}} \left(e^{-2g_I t_I V^I} \right)^{\hat{a}}_{\hat{c}} \Phi^{\hat{c}} \right\}^{2/3} \\ & - e^{3\sigma} \left[\int d^2\theta \Phi^{\hat{a}} d_{\hat{a}}^{\hat{b}} \rho_{\hat{b}\hat{c}} (\partial_y - 2ig_I t_I \Sigma^I)^{\hat{c}}_{\hat{d}} \Phi^{\hat{d}} + \text{h.c.} \right] \\ & + \mathcal{L}_{\text{vec}} + 2 \sum_{y_*=0,L} \mathcal{L}^{(y_*)} \delta(y - y_*), \end{aligned} \quad (2.2)$$

where $d_{\hat{a}}^{\hat{b}} = \text{diag}(\mathbf{1}_{2n_C}, -\mathbf{1}_{2n_H})$, $\rho_{\hat{a}\hat{b}} = i\sigma_2 \otimes \mathbf{1}_{n_C+n_H}$, $\bar{\Phi}_{\hat{b}} \equiv (\Phi^{\hat{b}})^\dagger$, and \mathcal{L}_{vec} is defined as

$$\mathcal{L}_{\text{vec}} \equiv \int d^2\theta \left\{ -\frac{\mathcal{N}_{IJ}(\Sigma)}{4} \mathcal{W}^I \mathcal{W}^J + \frac{\mathcal{N}_{IJK}}{48} \bar{D}^2 (V^I D^\alpha \partial_y V^J - D^\alpha V^I \partial_y V^J) \mathcal{W}_\alpha^K \right\} + \text{h.c.} \quad (2.3)$$

This contains the kinetic terms for the vector multiplets V^I and the Chern-Simons terms. The indices \hat{a}, \hat{b} run over the whole $2(n_C + n_H)$ chiral multiplets coming from the hypermultiplets. As mentioned in the previous subsection, we consider the case of $n_C = 1$ in the following. Here σ_2 in $\rho_{\hat{a}\hat{b}}$ acts on each hypermultiplet (Φ^{2a-1}, Φ^{2a}) . \mathcal{N} is a cubic polynomial called the norm function,⁶ which is defined by

$$\mathcal{N}(X) \equiv C_{IJK} X^I X^J X^K. \quad (2.4)$$

A real constant tensor C_{IJK} is completely symmetric for the indices, and $\mathcal{N}_I(X) \equiv \partial \mathcal{N} / \partial X^I$, $\mathcal{N}_{IJ}(X) \equiv \partial^2 \mathcal{N} / \partial X^I \partial X^J$, and so on. The superfield strength $\mathcal{W}_\alpha^I \equiv -\frac{1}{4} \bar{D}^2 D_\alpha V^I$ and $\mathcal{V}^I \equiv -\partial_y V^I + \Sigma^I + \bar{\Sigma}^I$ are gauge-invariant quantities. The generators t_I are anti-hermitian. For a gauge multiplet of a non-abelian gauge group G , the indices I, J run over $\dim G$ values and \mathcal{N}_{IJ} are common for them. The index a for the hypermultiplets are divided into irreducible representations of G . The fractional powers in the first line of (2.2) appear after integrating V_E out. The boundary Lagrangian $\mathcal{L}^{(y_*)}(y_* = 0, L)$ can be introduced independently of the bulk Lagrangian.

Note that (2.2) is a shorthand expression of the full SUGRA action. We can always restore the full action by the promotion,

$$\int d^4\theta \{ \cdots \} \rightarrow \frac{1}{2} [\cdots]_D, \quad \int d^2\theta \{ \cdots \} + \text{h.c.} \rightarrow [\cdots]_F, \quad (2.5)$$

where $[\cdots]_D$ and $[\cdots]_F$ denote the D - and F -term action formulae of the $N = 1$ superconformal formulation [16], which are compactly listed in Appendix C of Ref. [5].⁷ Here we omitted the spacetime integral. This promotion restores the dependence of the action on the components of the $N = 1$ Weyl multiplet E_W , such as the Einstein-Hilbert term and the gravitino-dependent terms. The Weyl multiplet also contains some auxiliary fields. After integrating them out, some terms in (2.2) are modified. Practically, the kinetic terms for V^I are the only such terms that are relevant to the phenomenological discussions. Their

⁶ This corresponds to the prepotential in the $N = 2$ global SUSY case.

⁷ The superfield descriptions of the D - and F -term formulae on the ordinary superspace are provided at the linear order in the fields belonging to E_W in Ref. [17].

kinetic functions are read off as $-\mathcal{N}_{IJ}(\Sigma)/4$ from the first term in (2.3). This will be modified after integrating out the above-mentioned auxiliary fields, and the correct kinetic function is obtained as $\{\mathcal{N}_{a_{IJ}}\}(\Sigma)/2$, where [4]

$$a_{IJ} \equiv -\frac{1}{2\mathcal{N}} \left(\mathcal{N}_{IJ} - \frac{\mathcal{N}_I \mathcal{N}_J}{\mathcal{N}} \right). \quad (2.6)$$

2.4 Superconformal gauge-fixing in 5D

Here we provide some comments on the superconformal gauge-fixing in 5D SUGRA. Since we will not impose the gauge-fixing conditions at the 5D stage,⁸ the readers can skip this subsection. Nevertheless, the comments presented here may help the readers to understand the structure of 5D SUGRA set-up.

The Lagrangian (2.2) with the promotion (2.5) is invariant (up to total derivatives) under the superconformal symmetries. In order to obtain the usual Poincaré SUGRA, we have to eliminate the extra symmetries by imposing the gauge-fixing conditions. A conventional choice of such conditions is expressed in our $N = 1$ superfield notation as

$$\begin{aligned} \mathcal{N} \left(\frac{\mathcal{V}}{V_E} \right) \Big|_0 &= d_{\hat{a}}^{\hat{b}} \bar{\Phi}_{\hat{b}} \Phi^{\hat{a}} \Big|_0 = M_5^3, \\ \mathcal{N} \left(\frac{\mathcal{V}}{V_E} \right) \Big|_{\theta} &= \mathcal{N} \left(\frac{\mathcal{V}}{V_E} \right) \Big|_{\bar{\theta}^2 \theta} = d_{\hat{a}}^{\hat{b}} \bar{\Phi}_{\hat{b}} \Phi^{\hat{a}} \Big|_{\theta} = \Phi^{\hat{a}} d_{\hat{a}}^{\hat{b}} \rho_{\hat{b}\hat{c}} \Phi^{\hat{c}} \Big|_{\theta} = 0, \quad \dots, \end{aligned} \quad (2.7)$$

where M_5 is the 5D Planck mass, and the symbols $|_0$, $|\theta$ and $|\bar{\theta}^2 \theta$ denote the lowest, θ - and $\bar{\theta}^2 \theta$ -components in the superfields, respectively. $V_E = e_y^4 + \theta \psi_y^+ + \bar{\theta} \bar{\psi}_y^+ + \dots$ is the real general multiplet coming from the 5D Weyl multiplet (see Table 1), where ψ_y^+ is the Z_2 -even 5th-component of the gravitino. The conditions in the first line fix the dilatation, and those in the second line fix the conformal SUSY. They reproduce the Einstein-Hilbert term $\mathcal{L} = -\frac{M_5^3}{2} e^{(5)} \mathcal{R}^{(5)} + \dots$, where $e^{(5)}$ is the determinant of the fünfbein, $\mathcal{R}^{(5)}$ is the 5D Ricci scalar, from the D -term action formula.

The conditions in (2.7) indicate that there is one multiplet whose components are not physical in each of the vector and hypermultiplet sectors. Such a multiplet is the graviphoton multiplet in the vector multiplet sector, and the compensator multiplet in the hypermultiplet sector. However, the graviphoton B_M ($M = \mu, y$) itself is exceptional. Since (2.7) does not involve the vector components, the graviphoton is always physical. The first condition in (2.7) suggests that $\Sigma^{I'}|_0$ generically have nonvanishing VEVs. (Since $\Sigma^{I''}$ are Z_2 -odd, they do not have zero-modes nor VEVs.) Specifically, the lowest components of V^I and Σ^I are

$$V^I = \theta \sigma^\mu \bar{\theta} W_\mu^I + \dots, \quad \Sigma^I = \frac{1}{2} (e_y^4 M^I - i W_y^I) + \dots, \quad (2.8)$$

where M^I and W_M^I are the real scalar and vector components of the 5D vector multiplet \mathbf{V}^I , and the 4D vector part of the graviphoton multiplet is identified as

$$V_B \equiv \frac{\mathcal{N}_I}{3\mathcal{N}} (2\text{Re} \langle \Sigma \rangle) V^I, \quad (2.9)$$

⁸ The superconformal gauge-fixing will be imposed after 4D effective action is derived because it breaks the $N = 1$ off-shell structure of the action.

whose fermionic component vanishes by (2.7). The corresponding chiral multiplet part is defined as

$$\mathcal{T} \equiv \frac{\mathcal{N}_I}{3\mathcal{N}}(2\text{Re}\langle\Sigma\rangle)\Sigma^I. \quad (2.10)$$

In contrast to V_B , this remains physical under the gauge-fixing conditions. In fact, (2.7) and (2.9) suggest that

$$\mathcal{T} = \frac{1}{2}(e_y^4 - iB_y) + \theta\psi_y^+ + \dots, \quad (2.11)$$

where $B_M = \frac{\mathcal{N}_I}{3\mathcal{N}}(2\text{Re}\langle\Sigma\rangle)W_M^I$ is the graviphoton. We have chosen the coordinate y such that $\langle e_y^4 \rangle = 1$. Eq.(2.11) shows that \mathcal{T} no longer belong to the vector multiplet sector after the gauge-fixing (2.7), but it is the “5D radion multiplet” belonging to the 5D gravitational sector.

Therefore, among Σ^I , one combination \mathcal{T} has a different origin from the others. We can also see this fact explicitly from the action. The condition (2.7) suggests that Φ^2 must have a nonzero VEV, and it plays a similar role to the chiral compensator multiplet in 4D SUGRA. (Φ^1 does not have a VEV because it is Z_2 -odd.) To emphasize this point, we rewrite the hypermultiplets as $\phi \equiv (\Phi^2)^{2/3}$ and $\hat{\Phi}^{\hat{a}} \equiv \Phi^{\hat{a}}/\Phi^2$ ($\hat{a} \neq 2$) so that their Weyl weights are one and zero, respectively. Then the first line of (2.2) is expanded around $\langle\Sigma^I\rangle$ as

$$\begin{aligned} \mathcal{L}_D &= -3\hat{\mathcal{N}}^{1/3}(\langle\mathcal{V}\rangle) \int d^4\theta |\phi|^2 \left\{ 1 + \frac{\mathcal{N}_I}{\mathcal{N}}(\langle\mathcal{V}\rangle)\tilde{\mathcal{V}}^I + \frac{3\mathcal{N}\mathcal{N}_{IJ} - 2\mathcal{N}_I\mathcal{N}_J}{18\mathcal{N}^2}(\langle\mathcal{V}\rangle)\tilde{\mathcal{V}}^I\tilde{\mathcal{V}}^J \right\} + \dots \\ &= -\hat{\mathcal{N}}^{1/3}(2\text{Re}\langle\Sigma\rangle) \int d^4\theta |\phi|^2 \left\{ 3(\mathcal{T} + \bar{\mathcal{T}}) - (a \cdot \mathcal{P})_{IJ} \tilde{\mathcal{V}}^I \tilde{\mathcal{V}}^J + \mathcal{O}(\hat{\Phi}^2, \tilde{\mathcal{V}}^3) \right\}, \end{aligned} \quad (2.12)$$

where $\tilde{\mathcal{V}}^I = -\partial_y V^I + \tilde{\Sigma}^I + \tilde{\tilde{\Sigma}}$ denotes the fluctuation part of \mathcal{V}^I , and \mathcal{P}^I_J is a projection operator defined as

$$\mathcal{P}^I_J(\mathcal{X}) \equiv \delta^I_J - \frac{\mathcal{X}^I \mathcal{N}_J}{3\mathcal{N}}(\mathcal{X}), \quad (2.13)$$

which has a property,

$$\{\mathcal{N}_I \mathcal{P}^I_J\}(\mathcal{X}) = \mathcal{P}^I_J(\mathcal{X}) \mathcal{X}^J = 0. \quad (2.14)$$

The argument of $(a \cdot \mathcal{P})_{IJ} \equiv a_{IK} \mathcal{P}^K_J$ is $(2\text{Re}\langle\Sigma\rangle)$. In the second line of (2.12), we can see that the first term has the no-scale structure peculiar to the 5D radion multiplet [18], and the second term does not include \mathcal{T} due to the projection operator \mathcal{P}^I_J . Hence Σ^I are divided into two categories according to their origins.

2.5 Gauging and mass scales

In SUGRA, an introduction of any mass scales into the action requires gauging some of the isometries on the hyperscalar manifold by the moduli multiplets $\mathbf{V}^{I'}$, *i.e.*, we have to deal with the gauged SUGRA. For example, the bulk cosmological constant is induced when the compensator hypermultiplet (Φ^1, Φ^2) is charged, and a bulk mass parameter for a physical hypermultiplet is generated when it is charged for $\mathbf{V}^{I'}$. The gauging by $\mathbf{V}^{I''}$ leads to the usual gauging by a 4D massless gauge multiplet in 4D effective theory. We omit the latter

type of gauging in the following expressions because it does not play a significant role in the derivation of the effective theory and can be easily restored in the 4D effective action. We assume that all the gaugings by $\mathbf{V}^{I'}$ are abelian, and are chosen to σ_3 -direction in the (Φ^{2a-1}, Φ^{2a}) -space, for simplicity. Thus, the generators and the gauge couplings are chosen as

$$(ig_{I'} t_{I'})^{\hat{a}}_{\hat{b}} = \sigma_3 \otimes \text{diag} \left(\frac{3}{2} k_{I'}, c_{2I'}, c_{3I'}, \dots, c_{(n_H+1)I'} \right), \quad (2.15)$$

where σ_3 acts on each hypermultiplet (Φ^{2a-1}, Φ^{2a}) . Note that these coupling constants are Z_2 -odd. Such kink-type couplings can be realized in SUGRA context by the mechanism proposed in Ref. [19]. In contrast to Ref. [13], the compensator multiplet also has non-vanishing charges, which lead to the warping of the 5D spacetime.

Then, after rescaling chiral multiplets by a factor $e^{3\sigma/2}$, we obtain

$$\begin{aligned} \mathcal{L} = & -3 \int d^4\theta \mathcal{N}^{1/3}(\mathcal{V}) \left\{ e^{-3k \cdot V} |\Phi^1|^2 + e^{3k \cdot V} |\Phi^2|^2 \right. \\ & \left. - \sum_{a=2}^{n_H+1} \left(e^{-2c_a \cdot V} |\Phi^{2a-1}|^2 + e^{2c_a \cdot V} |\Phi^{2a}|^2 \right) \right\}^{2/3} \\ & - 2 \left[\int d^2\theta \left\{ \Phi^1 (\partial_y + 3k \cdot \Sigma) \Phi^2 - \sum_{a=2}^{n_H+1} \Phi^{2a-1} (\partial_y + 2c_a \cdot \Sigma) \Phi^{2a} \right\} + \text{h.c.} \right] \\ & + \mathcal{L}_{\text{vec}} + 2 \sum_{y_*=0, L} \left[\int d^2\theta (\Phi^2)^2 W^{(y_*)} (\hat{\Phi}^{2a}) + \text{h.c.} \right] \delta(y - y_*), \end{aligned} \quad (2.16)$$

where $k \cdot V \equiv \sum_{I'} k_{I'} V^{I'}$, $c_a \cdot V \equiv \sum_{I'} c_{aI'} V^{I'}$, and $\hat{\Phi}^{2a} = \Phi^{2a}/\Phi^2$. For simplicity, we have introduced only superpotentials $W^{(y_*)}$ in the boundary Lagrangians. The hypermultiplets appear in $W^{(y_*)}$ only through $\hat{\Phi}^{2a}$ because physical chiral multiplets must have zero Weyl weights in $N = 1$ superconformal formulation [16] and Φ^{2a-1} vanish at the boundaries due to their orbifold parities. The boundary Lagrangians can also depend on boundary-localized 4D superfields, which are not considered in this article. The power of Φ^2 in front of $W^{(y_*)}$ is determined by the requirement that the argument of the F -term action formula must have the Weyl weight 3. (The Weyl weight of $\Phi^{\hat{a}}$ is $3/2$.)

3 4D effective theory and its properties

3.1 4D effective Lagrangian

Following the off-shell dimensional reduction developed in Ref. [6], we can derive the 4D effective action, keeping the $N = 1$ off-shell structure. A detailed derivation is summarized in Appendix A. The result is

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & -\frac{1}{4} \left[\int d^2\theta \sum_r f_{\text{eff}}^r(T) \text{tr} (\mathcal{W}^r \mathcal{W}^r) + \text{h.c.} \right] \\ & + \int d^4\theta |\phi|^2 \Omega_{\text{eff}}(|Q|^2, \text{Re } T) + \left[\int d^2\theta \phi^3 W_{\text{eff}}(Q, T) + \text{h.c.} \right], \end{aligned} \quad (3.1)$$

where the gauge multiplets are summarized in the matrix forms for the non-abelian gauge groups, the index r indicates the gauge sectors, and \mathcal{W}^r is the field strength supermultiplet for a massless 4D vector multiplet V^r . Q_a ($a \geq 2$) and $T^{I'}$ are the zero-modes for $\hat{\Phi}^{2a}$ and $\Sigma^{I'}$, respectively. We have used the same symbols for the zero-modes V^r as the corresponding 5D multiplets. Each function in (3.1) is expressed as

$$\begin{aligned}
f_{\text{eff}}^r(T) &= \sum_{I'} \xi_{I'}^r T^{I'}, \\
\Omega_{\text{eff}}(|Q|^2, \text{Re } T) &\equiv -3e^{-K_{\text{eff}}/3} \\
&= \hat{\mathcal{N}}^{1/3}(\text{Re } T) \left[-3Y(k \cdot T) + 2 \sum_a Y((k + d_a) \cdot T) |Q_a|^2 \right. \\
&\quad \left. + \sum_{a,b} \tilde{\Omega}_{a,b}^{(4)}(\text{Re } T) |Q_a|^2 |Q_b|^2 + \mathcal{O}((k \cdot \mathcal{P})^2) + \mathcal{O}(|Q|^6) \right], \\
W_{\text{eff}}(Q, T) &= W^{(0)}(Q) + e^{-3k \cdot T} W^{(L)}(e^{-d_a \cdot T} Q_a), \tag{3.2}
\end{aligned}$$

where $\xi_{I'}^r$ are real constants determined from $C_{I'J''K''}$, $d_{aI'} \equiv c_{aI'} - \frac{3}{2}k_{I'}$, and

$$Y(z) \equiv \frac{1 - e^{-2\text{Re } z}}{2\text{Re } z}. \tag{3.3}$$

The functions $\tilde{\Omega}_{a,b}^{(4)}$ are defined as

$$\tilde{\Omega}_{a,b}^{(4)} \equiv - \frac{(d_a \cdot \mathcal{P} a^{-1} \cdot d_b) \left\{ Y((k + d_a + d_b) \cdot T) - \frac{Y(d_a \cdot T)Y(d_b \cdot T)}{Y(-k \cdot T)} \right\}}{\{(k + d_a) \cdot \text{Re } T\} \{(k + d_b) \cdot \text{Re } T\}} + \frac{Y((k + d_a + d_b) \cdot T)}{3}. \tag{3.4}$$

In the derivation of Ω_{eff} summarized in Appendix (A.2), we have assumed that

$$k_{I'} \mathcal{P}^{I'}_{J'}(\text{Re } T) = 0, \tag{3.5}$$

in order to obtain an analytic expression. Thus we focus on a case that the moduli VEVs are (at least approximately) aligned to satisfy (3.5) by some mechanism. When the number of the moduli is two, the above effective Lagrangian reduces to that in Ref. [13] in the limit of $k_{I'} \rightarrow 0$.⁹ The constraint (3.5) disappears in this limit.

3.2 Superconformal gauge-fixing and mass dimension

Here we mention the superconformal gauge-fixing in 4D SUGRA. Since the effective action (3.1) (with the promotion (2.5)) has the $N = 1$ superconformal symmetries, we have to impose the gauge-fixing conditions in order to obtain the usual Poincaré SUGRA. The extra symmetries to eliminate are the dilatation \mathbf{D} , the $U(1)_A$ automorphism, the

⁹ There are typos in Ref. [13]. The indices of the derivatives of the norm functions should be replaced as $\hat{\mathcal{N}}_1 \leftrightarrow \hat{\mathcal{N}}_2$ and $\hat{\mathcal{N}}_{11} \leftrightarrow \hat{\mathcal{N}}_{22}$ in (2.15) and Sec.3.1 of Ref. [13].

conformal SUSY \mathbf{S} , and the conformal boost \mathbf{K} . For a real general multiplet $U = C + \theta\zeta + \bar{\theta}\bar{\zeta} + \dots + \frac{1}{2}\theta^2\bar{\theta}^2 D$, the D -term action formula is written as [5, 16]

$$[U]_D = e^{(4)} \left\{ D + \frac{1}{3}C \left(\mathcal{R}^{(4)} - 4\epsilon^{\mu\nu\rho\tau} \bar{\psi}_\mu \bar{\sigma}_\nu \partial_\rho \psi_\tau \right) + \frac{4i}{3} \zeta \sigma^{\mu\nu} \partial_\mu \psi_\nu + \dots \right\}, \quad (3.6)$$

where $e^{(4)}$ is the determinant of the vierbein, $\mathcal{R}^{(4)}$ is the 4D Ricci scalar, and $\sigma^{\mu\nu} \equiv \frac{1}{4}(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu)$.¹⁰ We omitted the spacetime integral. A conventional choice of the gauge-fixing is as follows. The \mathbf{D} -gauge is fixed so that the Einstein-Hilbert term is realized, and the \mathbf{S} -gauge is fixed so that the kinetic mixing between the matter fermions and the gravitino is absent. The corresponding gauge-fixing conditions are given by

$$(|\phi|^2 \Omega_{\text{eff}})|_0 = -3M_{\text{Pl}}^2, \quad (|\phi|^2 \Omega_{\text{eff}})|_\theta = 0. \quad (3.7)$$

The $U(1)_A$ -symmetry is eliminated by fixing the phase of $\phi|_0$ to zero. The \mathbf{K} -gauge fixing condition is irrelevant to the current discussion. The resultant Lagrangian in the gravitational sector is obtained as

$$\mathcal{L}_{\text{gravi}} = -\frac{M_{\text{Pl}}^2}{2} e^{(4)} \left(\mathcal{R}^{(4)} - 4\epsilon^{\mu\nu\rho\tau} \bar{\psi}_\mu \bar{\sigma}_\tau \partial_\nu \psi_\rho \right) + \dots. \quad (3.8)$$

We comment on the relations to the 5D gauge-fixing we chose in (2.7). From the definitions of $T^{I'}$ and ϕ in Appendix A.1, the \mathbf{D} -gauge fixing in (2.7) corresponds to the following conditions in 4D effective theory.

$$\begin{aligned} \hat{\mathcal{N}}^{1/3}(\text{Re} \langle T \rangle) &= \int_0^L dy \mathcal{N}^{1/3}(2\text{Re} \langle \Sigma \rangle) = M_5 L, \\ \langle \phi \rangle &= (\langle \Phi^2 \rangle)^{2/3} \simeq M_5. \end{aligned} \quad (3.9)$$

We have assumed that VEVs of all the physical hypermultiplets are much smaller than $M_{\text{Pl}}^{3/2}$, and used the facts that $\langle \Sigma^{I''} \rangle = 0$ and all $\langle \Sigma^{I'} \rangle$ have the same y -dependence under the condition (3.5). Namely we can read off the 5D scales M_5 and L^{-1} from (3.9).

Before the gauge fixing, all quantities in the action do not have the mass dimension. It can be defined after the \mathbf{D} -gauge fixing that introduces the mass scale into the theory. Since \mathbf{D} corresponds to the scale transformation, the mass dimension seems to be identified with the \mathbf{D} charge, *i.e.*, the Weyl weight. However, they are completely different as shown in Table 2. For instance, the former assigns a nonzero value to the coordinate x^μ while the latter does not. The mass dimensions of the gravitational fields are determined from (3.8). As for the chiral and vector multiplets, the numbers in the table denote those for the lowest components. They increase for higher components by 1/2. The mass dimension of the moduli $T^{I'}$ is determined so that their VEVs have a dimension of length. Note that the gauge-fixing conditions break the $N = 1$ off-shell structure and the theory cannot be expressed in terms of superfields any longer. In order to express the action in terms of the

¹⁰ We follow the spinor notation of Ref. [20].

	x^μ	θ^α	$e_\mu{}^\nu$	ψ_μ^α	Q_a	$T^{I'}$	$V^{I''}$
Weyl weight	0	$(-1/2)$	-1	$-1/2$	0	0	0
mass dimension	-1	$-1/2$	0	$1/2$	1	-1	0

Table 2: The Weyl weights and the mass dimensions of the coordinates and fields in 4D effective theory.

component fields that have the mass dimensions listed in Table 2, we rescale each quantity as

$$\begin{aligned}
x^\mu &\rightarrow M_{\text{Pl}} x^\mu, \\
(e_\mu{}^\nu, \psi_\mu^\alpha) &\rightarrow \frac{1}{M_{\text{Pl}}} (e_\mu{}^\nu, \psi_\mu^\alpha), \\
(Q_a, T^{I'}, V^{I''}) &\rightarrow \left(\frac{Q_a}{M_{\text{Pl}}}, M_{\text{Pl}} T^{I'}, V^{I''} \right).
\end{aligned} \tag{3.10}$$

Then the coupling constants $k_{I'}$ and $c_{I'}$ are accompanied with M_{Pl} in the rescaled action while $g_{I''}$ are not. So we also rescale these constants as

$$M_{\text{Pl}} k_{I'}, M_{\text{Pl}} c_{I'}, g_{I''} \rightarrow k_{I'}, c_{I'}, g_{I''}. \tag{3.11}$$

Hence the moduli couplings $k_{I'}$ and $c_{I'}$ are regarded as mass parameters while the gauge couplings $g_{I''}$ are dimensionless constants. After this procedure, M_{Pl} appears only in (3.8) and the gravitational interactions.

3.3 Moduli kinetic terms

Since the moduli VEVs satisfy (3.5), the combination $k \cdot T$ is expressed as

$$k \cdot T = \kappa T_{\text{rad}}, \tag{3.12}$$

where

$$\kappa \equiv \frac{1}{L} (k \cdot \text{Re} \langle T \rangle), \quad T_{\text{rad}} \equiv L \frac{\hat{\mathcal{N}}_{I'}}{3\hat{\mathcal{N}}} (\text{Re} \langle T \rangle) T^{I'}. \tag{3.13}$$

We have determined the coefficient so that $\text{Re} \langle T_{\text{rad}} \rangle = L$. Note that T_{rad} is identified with the zero-mode of the 5D radion multiplet \mathcal{T} defined in (2.10). Since $\hat{\mathcal{N}}^{1/3}(\text{Re} T)$ is expanded around $T = \langle T \rangle$ as

$$\begin{aligned}
\hat{\mathcal{N}}^{1/3}(\text{Re}(\langle T \rangle + \delta T)) &= \hat{\mathcal{N}}^{1/3}(\text{Re} \langle T \rangle) \left\{ 1 + \frac{\hat{\mathcal{N}}_{I'}}{3\hat{\mathcal{N}}} (\text{Re} \langle T \rangle) \text{Re} \delta T^{I'} + \mathcal{O}(\delta T^2) \right\} \\
&= \frac{1}{L} \hat{\mathcal{N}}^{1/3}(\text{Re} \langle T \rangle) \text{Re} T_{\text{rad}} + \mathcal{O}(\delta T^2),
\end{aligned} \tag{3.14}$$

Ω_{eff} in (3.2) becomes

$$\Omega_{\text{eff}} = -\frac{3}{L}\hat{\mathcal{N}}^{1/3}(\text{Re}\langle T \rangle)\frac{1 - e^{-2\kappa\text{Re}T_{\text{rad}}}}{2\kappa} + \mathcal{O}(\delta T^2) + \dots, \quad (3.15)$$

where the ellipsis denotes terms involving other multiplets than T_{rad} . This is the kinetic term of the radion multiplet in the Randall-Sundrum spacetime [21]. Thus, the alignment of moduli VEVs in (3.5) is interpreted as the condition for the spacetime to be the Randall-Sundrum spacetime, and κ defined in (3.13) is identified with the AdS curvature scale that is related to the bulk cosmological constant. After the gauge-fixing (3.7), we obtain

$$M_{\text{Pl}}^2 = -\frac{1}{3}\langle |\phi|^2 \Omega_{\text{eff}} \rangle \simeq -\frac{1}{3}M_5^2 \cdot \left(-\frac{3}{L}\right) \cdot M_5 L \cdot \frac{1 - e^{-2\kappa L}}{2\kappa} = \frac{M_5^3 (1 - e^{-2\kappa L})}{2\kappa}. \quad (3.16)$$

We have used (3.9). Eq.(3.16) is the well-known relation in the Randall-Sundrum spacetime [2].

The other moduli are orthogonal to T_{rad} in the moduli space, When there exist n_V moduli, they are parametrized by the coordinate system $\{\varphi^i\}$ ($i = 1, \dots, n_V - 1$) on the $(n_V - 1)$ -dimensional submanifold determined by ¹¹

$$\hat{\mathcal{N}}(\text{Re} T(\varphi)) = \hat{\mathcal{N}}(\text{Re}\langle T \rangle). \quad (3.17)$$

Since $\hat{\mathcal{N}}_{I'}(\text{Re} T)\frac{\partial T^{I'}}{\partial \varphi^i} = 0$, $(k \cdot \mathcal{P})_{I'}$ is expressed in terms of φ^i as

$$(k \cdot \mathcal{P})_{I'} = -\left\{ \frac{(k \cdot \text{Re} T)\hat{\mathcal{N}}_{I'J'}}{3\hat{\mathcal{N}}} \right\}_{\varphi=0} \text{Re} \left(\frac{\partial T^{J'}}{\partial \varphi^i} \Big|_{\varphi=0} \varphi^i \right) + \mathcal{O}(\varphi^2). \quad (3.18)$$

The kinetic terms for φ^i are contained in the $\mathcal{O}((k \cdot \mathcal{P})^2)$ -terms in (3.2), which start from ¹²

$$\delta\Omega_{\text{eff}} = -\frac{1}{4}\left\{ \hat{\mathcal{N}}^{1/3} (k \cdot \mathcal{P})_{I'} a^{I'J'} (k \cdot \mathcal{P})_{J'} \right\} (\text{Re} T) + \mathcal{O}(k^4, |Q|^2). \quad (3.19)$$

3.4 Quadratic terms in Ω_{eff} and Yukawa hierarchy

The coefficients of $|Q_a|^2$ in Ω_{eff} are important for generating the fermion mass hierarchy. The Yukawa couplings can be introduced only in the boundary actions due to the $N = 2$ SUSY in the bulk. We assume that they are contained in $W^{(0)}$ at $y = 0$,

$$W^{(0)} = \sum_{a,b,c} \lambda_{abc} \hat{\Phi}^{2a} \hat{\Phi}^{2b} \hat{\Phi}^{2c} + \dots, \quad (3.20)$$

where λ_{abc} are the holomorphic Yukawa coupling constants and are supposed to be of $\mathcal{O}(1)$. Then the effective theory has the Yukawa couplings,

$$W_{\text{eff}} = \sum_{a,b,c} \lambda_{abc} Q_a Q_b Q_c + \dots. \quad (3.21)$$

¹¹ One choice of $\{\varphi^i\}$ is $\varphi^i \equiv T^{I'=i} - \langle T^{I'=i} \rangle$. In this case, T^{n_V} becomes a function of φ^i through (3.17).

¹² This can be calculated by the perturbative expansion of the first equation in (A.28) in terms of $|k| \equiv (\sum_{I'} k_{I'}^2)^{1/2}$.

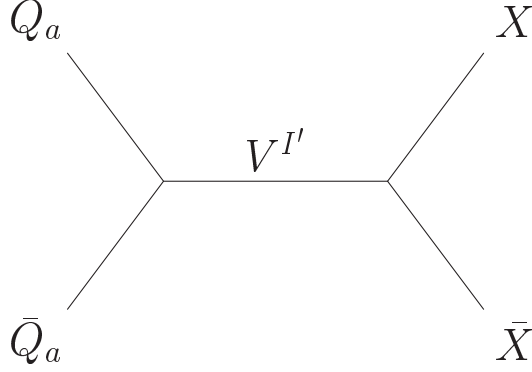


Figure 1: Feynmann diagrams contributing to $\tilde{\Omega}_{a,X}^{(4)}$ in the multi moduli case.

The physical Yukawa couplings y_{abc} are obtained by the canonical normalization of the chiral superfields Q_a , and we have

$$y_{abc} = \frac{\lambda_{abc}}{\sqrt{\langle Y_a Y_b Y_c \rangle}}, \quad (3.22)$$

where

$$Y_a \equiv 2\hat{\mathcal{N}}^{1/3}(\text{Re } T) \left\{ Y((k + d_a) \cdot T) + \tilde{\Omega}_{a,X}^{(4)}(\text{Re } T)|X|^2 + \mathcal{O}(|X|^4) \right\}. \quad (3.23)$$

The function $Y(z)$ is always positive, and approximated as

$$Y(z) \simeq \begin{cases} \frac{1}{2\text{Re } z}, & \text{Re } z > 0 \\ \frac{1}{2|\text{Re } z|} \exp \{2|\text{Re } z|\}, & \text{Re } z < 0 \end{cases} \quad (3.24)$$

From the 5D viewpoint, the wave function of Q_a is localized toward $y = 0$ ($y = L$) in the case that $(k + d_a) \cdot \langle \text{Re } T \rangle$ is positive (negative). As we can see from (3.22), y_{abc} is of $\mathcal{O}(1)$ when all the relevant fields are localized toward $y = 0$, while it is exponentially small when there is a field localized toward $y = L$ among them. This is the well-known split fermion mechanism [1].

3.5 Quartic couplings in Ω_{eff} and soft SUSY-breaking masses

The coefficients of $|Q_a|^2|Q_b|^2$ have a peculiar form to 5D SUGRA. This type of terms are important because they lead to the soft SUSY-breaking masses for the sfermions when Q_a and Q_b are identified with the quark or lepton superfield and the SUSY-breaking superfield X , respectively. Notice that the first term in (3.4) is absent in the single modulus case due to the projection operator $\mathcal{P}^{I'_{J'}}$. It is induced by integrating out the vector multiplets $V^{I'}$, which are the $N = 2$ partners of the moduli multiplets $\Sigma^{I'}$ [13]. The relevant Feynmann diagrams are depicted in Fig. 1. This can be seen from the fact that the coefficient of the first term in (3.4) involves the inverse matrix of $a_{I'J'}$, which comes from the propagator for $V^{I'}$. The existence of the projection operator $\mathcal{P}^{I'_{J'}}$ indicates that the

graviphoton multiplet V_B defined in (2.9) does not contribute to $\tilde{\Omega}_{a,b}^{(4)}$. This can be understood from the fact that most of the components of V_B are auxiliary fields, as mentioned in Sec. 2.4.

In the next section, we will consider a case that the F -term of one chiral multiplet X in the effective theory provides the main source of SUSY breaking and $\langle X \rangle \simeq 0$. In such a case, the $|Q_a|^2|X|^2$ -term contributes to the soft SUSY-breaking mass for the scalar component of Q_a , and it is expressed as (see (4.14) in Sec. 4.2)

$$m_a^2 \simeq -|F^X|^2 \frac{\tilde{\Omega}_{a,X}^{(4)}(\text{Re} \langle T \rangle)}{Y((k+d_a) \cdot \langle T \rangle)}. \quad (3.25)$$

Let us first consider the single modulus case. In this case, $\tilde{\Omega}_{a,X}^{(4)}$ is always positive because the first term in (3.4) is absent. Thus the soft scalar masses in (3.25) become

$$m_a^2 \simeq -|F^X|^2 \frac{Y((\kappa+d_a+d_X)L)}{3Y((\kappa+d_a)L)}, \quad (3.26)$$

and are found to be tachyonic. These tachyonic masses can be saved by quantum effects in some cases. The soft masses in (3.26) are exponentially suppressed when $\kappa+d_a < 0$ and $\kappa+d_X > 0$. This corresponds to a case that the matter Q_a is localized around $y=L$ while X is around $y=0$. In such a case, quantum effects to the soft scalar masses become dominant and may lead to non-tachyonic masses. However the large top quark mass cannot be realized because the top Yukawa coupling is suppressed in that case. (Recall that the Yukawa couplings are localized at the $y=0$ boundary.)

This problem can be evaded in the multi moduli case. Let us consider a case that

$$d_a \cdot \text{Re} \langle T \rangle < -\kappa L < 0 < d_X \cdot \text{Re} \langle T \rangle. \quad (3.27)$$

In this case, the $y=0$ boundary is identified with the UV brane, and Q_a and X are localized around the IR and UV branes respectively since $(k+d_a) \cdot \text{Re} \langle T \rangle < 0$ and $(k+d_X) \cdot \text{Re} \langle T \rangle > 0$. VEV of $\tilde{\Omega}_{a,b}^{(4)}$ is approximately expressed as

$$\begin{aligned} \tilde{\Omega}_{a,X}^{(4)}(\text{Re} \langle T \rangle) &\simeq \frac{d_a \cdot \mathcal{P}a^{-1} \cdot d_X}{\{(k+d_a) \cdot \text{Re} \langle T \rangle\} \{(k+d_X) \cdot \text{Re} \langle T \rangle\}} \frac{Y(d_a \cdot \langle T \rangle)Y(d_X \cdot \langle T \rangle)}{Y(-\kappa L)} \\ &\simeq \frac{d_a \cdot \mathcal{P}a^{-1} \cdot d_X}{\{(k+d_a) \cdot \text{Re} \langle T \rangle\} \{(k+d_X) \cdot \text{Re} \langle T \rangle\}} \cdot \frac{-\kappa L e^{-2(k+d_a) \cdot \text{Re} \langle T \rangle}}{2(d_a \cdot \text{Re} \langle T \rangle)(d_X \cdot \text{Re} \langle T \rangle)}. \end{aligned} \quad (3.28)$$

Therefore, (3.25) becomes

$$m_a^2 \simeq -|F^X|^2 \frac{(d_a \cdot \mathcal{P}a^{-1} \cdot d_X)\kappa L}{(d_a \cdot \text{Re} \langle T \rangle)(d_X \cdot \text{Re} \langle T \rangle)\{(k+d_X) \cdot \text{Re} \langle T \rangle\}}. \quad (3.29)$$

The sign of m_a^2 now depends on the (truncated) norm function $\hat{\mathcal{N}}$ and the directions of the gauging for $V^{I'}$. In fact, we can always realize non-tachyonic soft masses for any choices of $\hat{\mathcal{N}}$ by choosing the directions of the gauging such that

$$\frac{d_a \cdot \mathcal{P}a^{-1} \cdot d_X}{(d_a \cdot \text{Re} \langle T \rangle)(d_X \cdot \text{Re} \langle T \rangle)} < 0. \quad (3.30)$$

Furthermore, if n_V -dimensional vectors \vec{d}_a point to the same direction,

$$\vec{d}_a \propto \vec{n}, \quad (3.31)$$

the soft masses m_a^2 become independent of the “flavor index” a . (The direction \vec{n} must not be parallel to \vec{k} , otherwise all m_a^2 vanish.) This opens up the possibility to solve the SUSY flavor problem. We will discuss this issue in the next section.

A similar result is also obtained in a case that

$$d_a \cdot \text{Re} \langle T \rangle < 0 < -\kappa L < d_X \cdot \text{Re} \langle T \rangle. \quad (3.32)$$

Conditions for obtaining non-tachyonic (and flavor-universal) soft masses are the same as (3.30) (and (3.31)). In this case, however, the $y = 0$ boundary becomes the IR brane, and the approximate expressions of the soft masses are suppressed from (3.29) by a factor $e^{-2\kappa L} \gg 1$.¹³

4 Illustrative model

Now we specify the model that realizes the set-up mentioned in Sec. 2.1, and show some phenomenological analysis.

4.1 Hidden (mediation) sector contents

In this paper we assume that a single chiral multiplet X originating from a 5D hypermultiplet is responsible for the spontaneous SUSY breaking. We do not specify the potential of the hidden sector X and moduli $T^{I'}$ chiral multiplets. There are various ways to stabilize the moduli, including the size of the extra dimension. In general, the mechanism for the moduli stabilization determines the F -terms of $T^{I'}$, and thus affects the mediation of SUSY breaking to the MSSM sector. Here we do not specify the moduli stabilization and SUSY breaking mechanism, and treat (VEVs of) F^X and $F^{T^{I'}}$ as free parameters,¹⁴ while VEV of (the lowest component of) X is assumed to be almost vanishing $\langle X \rangle \ll \langle T^{I'} \rangle \simeq \mathcal{O}(1)$ in the 4D Planck mass unit.¹⁵

Instead of the non-vanishing F -terms, $F^{T^{I'}}$ ($I' = 1, 2, 3$) and F^X , we use ratios of them $\alpha_{I'}$ and a typical scale of SUSY breaking M_{SB} in order to parametrize the soft SUSY breaking parameters. They are defined as

$$\begin{aligned} \alpha_{I'} &\equiv \frac{\mathcal{F}^{T^{I'}}}{\mathcal{F}^X}, & M_{\text{SB}} &\equiv |\mathcal{F}^X|, \\ \mathcal{F}^A &\equiv E^A_B F^B, \end{aligned} \quad (4.1)$$

¹³ The typical SUSY-breaking mass scale M_{SB} is also suppressed by the same factor in this case. So a ratio m_a/M_{SB} is not suppressed. (See Sec. 4.2.)

¹⁴ We do not consider the D -term SUSY breaking in this article.

¹⁵ Concrete moduli stabilization and SUSY breaking mechanisms were studied in our previous work [13] based on Ref. [24], which are also applicable here.

where E^A_B is the vielbein¹⁶ for the Kähler metric $K_{A\bar{B}} \equiv \partial_A \partial_{\bar{B}} K_{\text{eff}}$. Then the vacuum value of the scalar potential is written as

$$\begin{aligned} V &= \sum_A |\mathcal{F}^A|^2 - 3e^K |W|^2 \\ &= \left(1 + \sum_{I'=1}^3 |\alpha_{I'}|^2\right) M_{\text{SB}}^2 - 3m_{3/2}^2, \end{aligned} \quad (4.2)$$

where $m_{3/2} = \langle e^{K/2} |W| \rangle$ is the gravitino mass. The moduli stabilization at a SUSY breaking Minkowski minimum $\langle V \rangle \simeq 0$ required by the observation leads to a relation

$$m_{3/2} \simeq \sqrt{\frac{1}{3} \left(1 + \sum_{I'=1}^3 |\alpha_{I'}|^2\right)} M_{\text{SB}}. \quad (4.3)$$

The ratios $\alpha_{I'}$ parameterize contributions of “the moduli mediation” induced by $F^{T'}$ compared to that of “the direct mediation” induced by F^X . In the following analysis we mainly consider cases that

$$|\alpha_{I'}| = \mathcal{O}(1) \quad \text{or} \quad \mathcal{O}(1/(4\pi^2)). \quad (4.4)$$

The latter values can be realized by the numerical value of $1/\ln(M_{\text{Pl}}/\text{TeV})$ if the moduli $T^{I'}$ are stabilized by nonperturbative effects like gaugino condensations at a SUSY anti-de Sitter vacuum, which is uplifted to the almost Minkowski minimum by the vacuum energy of the (TeV scale) SUSY breaking sector [22, 23]. We can identify F^X with a source of the uplifting vacuum energy based on a scenario of F-term uplifting [24, 25].

For the phenomenological analysis, we consider a case with three Z_2 -odd $U(1)_{I'}$ vector multiplets $\mathbf{V}^{I'}$ in 5D, where $I' = 1, 2, 3$. They generate three moduli chiral multiplets

$$T^{I'} = (T^1, T^2, T^3), \quad (4.5)$$

in the 4D effective theory and we choose the (truncated) norm function as

$$\hat{\mathcal{N}}(\text{Re } T) = (\text{Re } T^1)(\text{Re } T^2)(\text{Re } T^3). \quad (4.6)$$

Then the matrix $a_{I'J'}$ defined in (2.6) becomes diagonal. Explicit forms of the Kähler metric $K_{A\bar{B}}$ and $a_{I'J'}$ are shown in Appendix C. In the flat case ($\kappa L = 0$), the Kähler metric also becomes diagonal.

For concreteness, we further assume the moduli VEVs as

$$(\text{Re } \langle T^1 \rangle, \text{Re } \langle T^2 \rangle, \text{Re } \langle T^3 \rangle) = (4, 4, 1). \quad (\text{in the } M_{\text{Pl}} \text{ unit}) \quad (4.7)$$

Then the condition to realize the Randall-Sundrum spacetime (3.5) fixes the gauging direction of the compensator multiplets as

$$(k_1, k_2, k_3) = \left(\frac{1}{4}, \frac{1}{4}, 1\right) k_3. \quad (4.8)$$

The value of k_3 is determined by the warp factor through $\kappa L = k \cdot \text{Re } \langle T \rangle = 3k_3$.

¹⁶ We use the same index for the flat and curved coordinates on the Kähler manifold to save characters.

Multiplet	$\mathcal{Q}_i, \mathcal{U}_i, \mathcal{D}_i, \mathcal{L}_i, \mathcal{E}_i$	$\mathcal{H}_u, \mathcal{H}_d$	X
R-charge	1/2	1	2

Table 3: $U(1)_R$ -charges of the chiral multiplets.

4.2 Visible sector contents and soft SUSY-breaking parameters

We assume that the visible sector consists of the following MSSM matter contents:

$$\begin{aligned}
(V_1, V_2, V_3) &: \text{gauge vector multiplets,} \\
(\mathcal{Q}_i, \mathcal{U}_i, \mathcal{D}_i) &: \text{quark chiral multiplets,} \\
(\mathcal{L}_i, \mathcal{E}_i) &: \text{lepton chiral multiplets,} \\
(\mathcal{H}_u, \mathcal{H}_d) &: \text{Higgs chiral multiplets,}
\end{aligned} \tag{4.9}$$

where V_1, V_2, V_3 denote the gauge multiplets for $U(1)_Y, SU(2)_L, SU(3)_C$ originating from $\mathbf{V}^{I''}$, and the others are chiral multiplets from 5D hypermultiplets. The index $i = 1, 2, 3$ denotes the generation.

We also assume an approximate global $U(1)_R$ -symmetry that is responsible for the dynamical SUSY breaking [26]. We assign the R-charge as shown in Table 3. It is supposed to be broken by the nonperturbative effects. Then, the holomorphic Yukawa couplings and the μ -term in the boundary superpotentials as well as the gauge kinetic functions are independent of X . We further assume that these terms exist only at the $y = 0$ boundary. The resulting gauge kinetic functions and the superpotential for the visible sector in the 4D effective theory are parametrized as

$$\begin{aligned}
f_{\text{eff}}^r(T) &= \sum_{I'} \xi_{I'}^r T^{I'}, \\
W_{\text{MSSM}} &= \zeta_0 \mathcal{H}_u \mathcal{H}_d + \lambda_{ij}^u \mathcal{H}_u \mathcal{Q}_i \mathcal{U}_j + \lambda_{ij}^d \mathcal{H}_d \mathcal{Q}_i \mathcal{D}_j + \lambda_{ij}^e \mathcal{H}_d \mathcal{L}_i \mathcal{E}_j,
\end{aligned} \tag{4.10}$$

where $r = 1, 2, 3$ for $U(1)_Y, SU(2)_L, SU(3)_C$, and $\xi_{I'}^r$ are real constants determined by the coefficients $C_{I'J''K''}$ in the norm function \mathcal{N} , while ζ_0 and $\lambda_{ij}^{u,d,e}$ are in general complex constants. After the canonical normalization, the μ parameter is expressed as

$$\mu = \left\langle \frac{\zeta_0}{\sqrt{\hat{\mathcal{N}}^{1/3}(\text{Re } T) Y(k \cdot T) Y_{\mathcal{H}_u} Y_{\mathcal{H}_d}}} \right\rangle = \left\langle \frac{\zeta_0}{\sqrt{2Y(\kappa L) Y_{\mathcal{H}_u} Y_{\mathcal{H}_d}}} \right\rangle, \tag{4.11}$$

where Y_a is defined in (3.23), and we have used (4.7) at the second equality. The physical Yukawa couplings are expressed as

$$y_{ij}^u = \frac{\lambda_{ij}^u}{\sqrt{\langle Y_{\mathcal{H}_u} Y_{\mathcal{Q}_i} Y_{\mathcal{U}_j} \rangle}}, \quad y_{ij}^d = \frac{\lambda_{ij}^d}{\sqrt{\langle Y_{\mathcal{H}_d} Y_{\mathcal{Q}_i} Y_{\mathcal{D}_j} \rangle}}, \quad y_{ij}^e = \frac{\lambda_{ij}^e}{\sqrt{\langle Y_{\mathcal{H}_d} Y_{\mathcal{L}_i} Y_{\mathcal{E}_j} \rangle}}. \tag{4.12}$$

The holomorphic Yukawa couplings λ_{ij}^x ($x = u, d, e$) are assumed to be of $\mathcal{O}(1)$. The hierarchical structure of the Yukawa couplings are obtained by choosing the moduli couplings $c_{aI'}$ as explained in Sec. 3.4.

The soft SUSY-breaking parameters in the MSSM are defined by

$$\begin{aligned} \mathcal{L}_{\text{soft}} = & - \sum_{Q_a} m_{Q_a}^2 |Q_a|^2 - \frac{1}{2} \sum_r M_r \text{tr} (\lambda^r \lambda^r) \\ & - \sum_{i,j} \left\{ B\mu H_u H_d + y_{i,j}^u A_{i,j}^u H_u \tilde{q}_i \tilde{u}_j + y_{i,j}^d A_{i,j}^d H_d \tilde{q}_i \tilde{d}_j + y_{i,j}^e A_{i,j}^e H_d \tilde{l}_i \tilde{e}_j + \text{h.c.} \right\}, \end{aligned} \quad (4.13)$$

where λ^r ($r = 1, 2, 3$) and $Q_a = H_{u,d}, \tilde{q}_i, \tilde{u}_i, \tilde{d}_i, \tilde{l}_i, \tilde{e}_i$ are the gauginos and the scalar components of $\mathcal{H}_{u,d}, \mathcal{Q}_i, \mathcal{U}_i, \mathcal{D}_i, \mathcal{L}_i, \mathcal{E}_i$, respectively. The fields are all *canonically normalized*. These parameters are induced by the formulae [23, 27],

$$\begin{aligned} M_r &= \langle F^A \partial_A \ln (\text{Re } f_{\text{eff}}^r) \rangle, \quad m_{Q_a}^2 = -\langle F^A \bar{F}^{\bar{B}} \partial_A \partial_{\bar{B}} \ln Y_{Q_a} \rangle, \\ B &= - \left\langle F^A \partial_A \ln \left(\frac{\zeta_0}{\hat{\mathcal{N}}^{1/3} (\text{Re } T) Y(k \cdot T) Y_{\mathcal{H}_u} Y_{\mathcal{H}_d}} \right) \right\rangle - m_{3/2} e^{i\varphi}, \\ A_{ij}^u &= \langle F^A \partial_A \ln (Y_{\mathcal{H}_u} Y_{\mathcal{Q}_i} Y_{\mathcal{U}_j}) \rangle, \quad A_{ij}^d = \langle F^A \partial_A \ln (Y_{\mathcal{H}_d} Y_{\mathcal{Q}_i} Y_{\mathcal{D}_j}) \rangle, \\ A_{ij}^e &= \langle F^A \partial_A \ln (Y_{\mathcal{H}_d} Y_{\mathcal{L}_i} Y_{\mathcal{E}_j}) \rangle, \end{aligned} \quad (4.14)$$

where $A, B = X, T^1, T^2, T^3$, and $\varphi \equiv \arg \langle W \rangle$.

Now we show the (squared) soft scalar masses $m_{Q_a}^2$ as functions of the charges for the Z_2 -odd vector multiplets $V^{I'}$ ($I' = 1, 2, 3$). Since $\langle Y_a \rangle$ in the above formulae are functions of $(k + d_a) \cdot \text{Re} \langle T \rangle$, we normalize each charge as

$$\tilde{c}_a^{I'} \equiv (k_{I'} + d_{aI'}) \text{Re} \langle T^{I'} \rangle = (c_{aI'} - k_{I'}/2) \text{Re} \langle T^{I'} \rangle, \quad (4.15)$$

without summations for the index I' . The soft scalar mass $m_{Q_a}^2$ varies exponentially over $\mathcal{O}(1)$ ranges of $\tilde{c}_a^{I'}$. Thus we plot a quantity defined as

$$\beta(m_{Q_a}^2) \equiv \log_{10} \frac{\sqrt{|m_{Q_a}^2|}}{M_{\text{SB}}}. \quad (4.16)$$

In Figs. 2 and 3, we assume that $\alpha_1 = \alpha_3 = 1/(4\pi^2)$, $\alpha_2 = 2/(4\pi^2)$. Namely, contributions of the moduli mediation are tiny. In Fig. 4, we assume that $\alpha_1 = \alpha_2 = 1$, $\alpha_3 = 2$, *i.e.*, contributions of the moduli mediation are comparable to that of the direct mediation. The charge assignment of X is chosen as $(\tilde{c}_X^1, \tilde{c}_X^2, \tilde{c}_X^3) = (\tilde{c}_X - \frac{2\kappa L}{3}, \frac{\kappa L}{3}, \frac{\kappa L}{3})$ in all figures, while that of Q_a is chosen as $(\tilde{c}_{Q_a}^1, \tilde{c}_{Q_a}^2, \tilde{c}_{Q_a}^3) = (\frac{\kappa L}{3}, \tilde{c}_a - \frac{2\kappa L}{3}, \frac{\kappa L}{3})$ in Figs. 2 and 4, and $(\tilde{c}_{Q_a}^1, \tilde{c}_{Q_a}^2, \tilde{c}_{Q_a}^3) = (\tilde{c}_a - \frac{2\kappa L}{3}, \frac{\kappa L}{3}, \frac{\kappa L}{3})$ in Fig. 3. The surface with (without) a mesh describes a region $m_{Q_a}^2 > 0$ ($m_{Q_a}^2 < 0$). Note that the above charge assignments satisfy (3.31) because they are rewritten as

$$\begin{aligned} (d_X^1, d_X^2, d_X^3) &= (d_X, 0, 0), \\ (d_{Q_a}^1, d_{Q_a}^2, d_{Q_a}^3) &= \begin{cases} (0, d_a, 0), & \text{(in Figs. 2 and 4)} \\ (d_a, 0, 0). & \text{(in Fig. 3)} \end{cases} \end{aligned} \quad (4.17)$$

where $d_X \equiv (\tilde{c}_X - \kappa L)/4$ and $d_a \equiv (\tilde{c}_a - \kappa L)/4$.

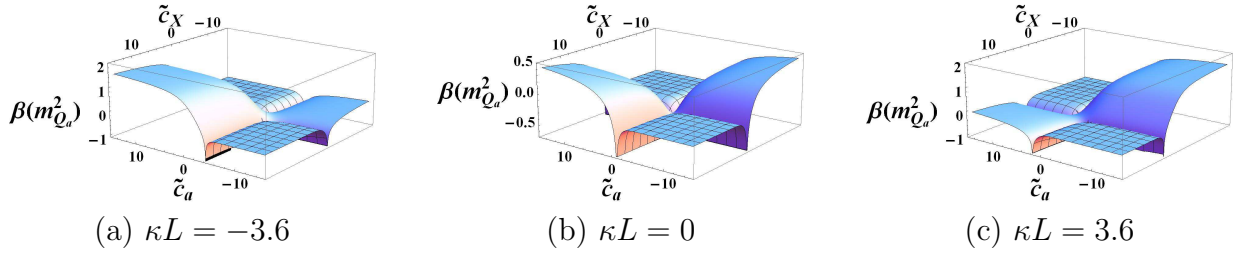


Figure 2: The charge dependences of $\beta(m_{Q_a}^2)$ defined in Eq.(4.16) with the norm function (4.6) and $\alpha_1 = \alpha_3 = 1/(4\pi^2)$, $\alpha_2 = 2/(4\pi^2)$. The charge assignment for Q_a and X is chosen as $(\tilde{c}_{Q_a}^1, \tilde{c}_{Q_a}^2, \tilde{c}_{Q_a}^3) = (\frac{\kappa L}{3}, \tilde{c}_a - \frac{2\kappa L}{3}, \frac{\kappa L}{3})$ and $(\tilde{c}_X^1, \tilde{c}_X^2, \tilde{c}_X^3) = (\tilde{c}_X - \frac{2\kappa L}{3}, \frac{\kappa L}{3}, \frac{\kappa L}{3})$. The surface with (without) a mesh describes the region $m_{Q_a}^2 > 0$ ($m_{Q_a}^2 < 0$).

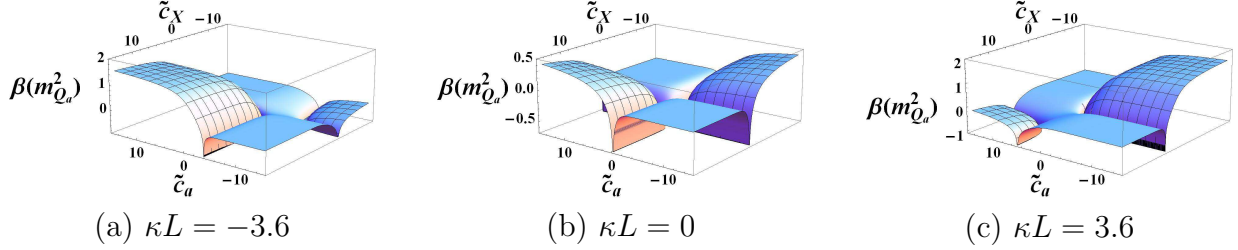


Figure 3: The charge dependences of $\beta(m_{Q_a}^2)$ defined in Eq.(4.16) with the norm function (4.6) and $\alpha_1 = \alpha_3 = 1/(4\pi^2)$, $\alpha_2 = 2/(4\pi^2)$. The charge assignment for Q_a and X is chosen as $(\tilde{c}_{Q_a}^1, \tilde{c}_{Q_a}^2, \tilde{c}_{Q_a}^3) = (\tilde{c}_a - \frac{2\kappa L}{3}, \frac{\kappa L}{3}, \frac{\kappa L}{3})$ and $(\tilde{c}_X^1, \tilde{c}_X^2, \tilde{c}_X^3) = (\tilde{c}_X - \frac{2\kappa L}{3}, \frac{\kappa L}{3}, \frac{\kappa L}{3})$. The surface with (without) a mesh describes the region $m_{Q_a}^2 > 0$ ($m_{Q_a}^2 < 0$).

Let us first consider a case that $\alpha_{I'} \ll 1$, *i.e.*, the direct mediation dominates. The soft scalar mass is expressed from (3.25) as

$$\frac{m_{Q_a}^2}{M_{\text{SB}}^2} \simeq -\frac{Y(\kappa L)\tilde{\Omega}_{Q_a, X}^{(4)}(\text{Re}\langle T \rangle)}{2Y(\tilde{c}_a)Y(\tilde{c}_X)}. \quad (4.18)$$

We can see that $|m_{Q_a}^2| > M_{\text{SB}}^2$ for $\tilde{c}_a\tilde{c}_X \gg 1$ in Figs. 2 and 3. This behavior can be understood from the fact that Q_a and X localize toward the same boundary in such a region. Especially in the warped case, the soft scalar mass is enhanced when they localize toward the IR boundary. On the other hand, they localize toward the opposite boundaries in a region $\tilde{c}_a\tilde{c}_X < -1$. Notice that $m_{Q_a}^2/M_{\text{SB}}^2$ is not exponentially suppressed and remains to be of $\mathcal{O}(1)$ in this region, because they are not sequestered due to the existence of the contact terms $|Q_a|^2|X|^2$ in Ω_{eff} induced by integrating out the heavy Z_2 -odd vector multiplets $V^{I'}$. This is in sharp contrast to the single modulus case where the sequestering occurs. Especially, in a region that $\tilde{c}_a\tilde{c}_X < -1$ and $(\tilde{c}_a - \kappa L)(\tilde{c}_X - \kappa L) < -1$, such induced contact terms dominate, and (4.18) can be approximated as

$$\frac{m_{Q_a}^2}{M_{\text{SB}}^2} \simeq -\frac{e^{2\kappa L}Y(\kappa L)}{2Y(-\kappa L)} \frac{d_{Q_a} \cdot \mathcal{P}a^{-1} \cdot d_X}{(d_{Q_a} \cdot \text{Re}\langle T \rangle)(d_X \cdot \text{Re}\langle T \rangle)} \simeq -\frac{d_{Q_a} \cdot \mathcal{P}a^{-1} \cdot d_X}{2(d_{Q_a} \cdot \text{Re}\langle T \rangle)(d_X \cdot \text{Re}\langle T \rangle)}. \quad (4.19)$$

Since the charge assignments of Q_a satisfy the condition (3.31), the flavor dependence of (4.19) cancels. With our choice of the norm function and the assumption (4.7), its

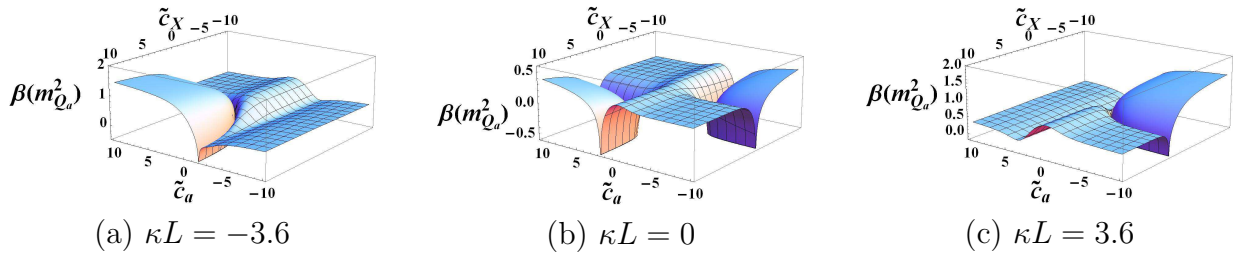


Figure 4: The charge dependences of $\beta(m_{Q_a}^2)$ defined in Eq.(4.16) with the norm function (4.6) and $\alpha_1 = \alpha_2 = 1$, $\alpha_3 = 2$. The charge assignment for Q_a and X is chosen as $(\tilde{c}_{Q_a}^1, \tilde{c}_{Q_a}^2, \tilde{c}_{Q_a}^3) = (\frac{\kappa L}{3}, \tilde{c}_a - \frac{2\kappa L}{3}, \frac{\kappa L}{3})$ and $(\tilde{c}_X^1, \tilde{c}_X^2, \tilde{c}_X^3) = (\tilde{c}_X - \frac{2\kappa L}{3}, \frac{\kappa L}{3}, \frac{\kappa L}{3})$. The surface with (without) a mesh describes the region $m_{Q_a}^2 > 0$ ($m_{Q_a}^2 < 0$).

approximate value is $16/3$ ($-32/3$) in the case of Fig. 2 (Fig. 3), irrespective of the value of κL . The non-tachyonic condition in such a flavor-universal region (3.30) is satisfied for the charge assignment of Fig. 2, while not for that of Fig. 3, as we can see the figures.

In a case that contributions from the moduli mediation are not negligible, *i.e.*, $|\alpha_{I'}| = \mathcal{O}(1)$, the above behaviors are disturbed. The expression of the soft scalar mass in such a case is given in (C.3). In the flat case ($\kappa L = 0$), for example, it is written as

$$\frac{m_{Q_a}^2}{M_{\text{SB}}^2} \simeq -\frac{\tilde{\Omega}_{Q_a, X}^{(4)}(\text{Re} \langle T \rangle)}{2Y(\tilde{c}_a)Y(\tilde{c}_X)} + \sum_{I'} \frac{|\alpha_{I'}|^2}{3} - \sum_{I', J'} \alpha_{I'} \bar{\alpha}_{J'} \tilde{c}_{Q_a}^{I'} \tilde{c}_{Q_a}^{J'} \mathcal{Y}(\tilde{c}_a), \quad (4.20)$$

where $\mathcal{Y}(x)$ is a function defined in (C.2). We have used the specific form of the norm function (4.6). In spite of the nontrivial \tilde{c}_a -dependence of the third term, there is still a region in which $m_{Q_a}^2$ is almost flavor-universal. This is due to the property of the function $\mathcal{Y}(x)$ that $x^2 \mathcal{Y}(x) \simeq 1$ for $|x| \gtrsim 3$.

By making use of the above properties, we construct a realistic model in the next subsection, and analyze the flavor structure of fermions and sfermions as well as the other phenomenological features. We comment that the boundary induced Kähler potentials $K^{(y_*)}$ ($y_* = 0, L$) are neglected in this paper. They may disturb the flavor structure if they dominate the contributions from the bulk. Here we just assume that such boundary contributions are small enough compared to those from the bulk.

4.3 Phenomenological analysis

In the following phenomenological analysis, the warp factor is chosen as

$$\kappa L = 3.6, \quad (4.21)$$

so that $e^{\kappa L} = \mathcal{O}(M_{\text{Pl}}/M_{\text{GUT}})$. This determines the compensator charges in (4.8) as $k_3 = 1.2$. The typical KK mass scale is set to the GUT scale,

$$M_{\text{KK}} \equiv \frac{\kappa \pi}{e^{\kappa L} - 1} = M_{\text{GUT}}. \quad (4.22)$$

This determines κ (and L) from (4.21). The 4D effective theory is valid below M_{KK} .

Here we comment on a consistency condition of our 5D setup. In order for the 5D description of the theory to be valid, the 5D curvature $\mathcal{R}^{(5)}$ must satisfy the condition $|\mathcal{R}^{(5)}| < M_5^2$ [28]. For the Randall-Sundrum spacetime, $\mathcal{R}^{(5)} = -20\kappa^2$. Thus, together with the relation (3.16) and the definition of M_{KK} , the consistency condition is rewritten as

$$e^{\kappa L} < \frac{\sqrt{2}\pi}{20^{3/4}} \frac{M_{\text{Pl}}}{M_{\text{KK}}} \simeq 0.47 \frac{M_{\text{Pl}}}{M_{\text{KK}}}. \quad (4.23)$$

For our choice of M_{KK} in (4.22), this indicates that $\kappa L < 4.0$, which is satisfied by (4.21).

In order to realize phenomenologically viable fermion and sfermion flavor structures, we assign the following $U(1)_{I'}$ charges for the Z_2 -odd vector multiplets $V^{I'}$ ($I' = 1, 2, 3$) to the MSSM matter multiplets $Q_a = (\mathcal{Q}_i, \mathcal{U}_i, \mathcal{D}_i, \mathcal{L}_i, \mathcal{E}_i, \mathcal{H}_u, \mathcal{H}_d)$. For the values of $\text{Re} \langle T^{I'} \rangle$ and $k_{I'}$ given by (4.7) and (4.8), the $U(1)_1$ charges are chosen as

$$\begin{aligned} \tilde{c}_{\mathcal{Q}_i}^{I'=1} &= (1.2, 1.2, 0.5), & \tilde{c}_{\mathcal{U}_i}^{I'=1} &= (1.2, 1.2, 0.5), & \tilde{c}_{\mathcal{D}_i}^{I'=1} &= (1.2, 1.2, 1.2), \\ \tilde{c}_{\mathcal{L}_i}^{I'=1} &= (1.2, 1.2, 1.2), & \tilde{c}_{\mathcal{E}_i}^{I'=1} &= (1.2, 1.2, 1.2), \\ \tilde{c}_{\mathcal{H}_u}^{I'=1} &= 1.0, & \tilde{c}_{\mathcal{H}_d}^{I'=1} &= 1.2, & \tilde{c}_X^{I'=1} &= 8.7, \end{aligned} \quad (4.24)$$

the $U(1)_2$ charges are assigned as

$$\begin{aligned} \tilde{c}_{\mathcal{Q}_i}^{I'=2} &= (-7.9, -5.9, 0), & \tilde{c}_{\mathcal{U}_i}^{I'=2} &= (-10.4, -5.9, 0), & \tilde{c}_{\mathcal{D}_i}^{I'=2} &= (-6.4, -6.9, -4.9), \\ \tilde{c}_{\mathcal{L}_i}^{I'=2} &= (-6.9, -6.9, -4.9), & \tilde{c}_{\mathcal{E}_i}^{I'=2} &= (-9.4, -3.9, -3.9), \\ \tilde{c}_{\mathcal{H}_u}^{I'=2} &= 0, & \tilde{c}_{\mathcal{H}_d}^{I'=2} &= -3.4, & \tilde{c}_X^{I'=2} &= 1.2, \end{aligned} \quad (4.25)$$

and the $U(1)_3$ charges are assigned as

$$\begin{aligned} \tilde{c}_{\mathcal{Q}_i}^{I'=3} &= (1.2, 1.2, 0), & \tilde{c}_{\mathcal{U}_i}^{I'=3} &= (1.2, 1.2, 0), & \tilde{c}_{\mathcal{D}_i}^{I'=3} &= (1.2, 1.2, 1.2), \\ \tilde{c}_{\mathcal{L}_i}^{I'=3} &= (1.2, 1.2, 1.2), & \tilde{c}_{\mathcal{E}_i}^{I'=3} &= (1.2, 1.2, 1.2), \\ \tilde{c}_{\mathcal{H}_u}^{I'=3} &= 0, & \tilde{c}_{\mathcal{H}_d}^{I'=3} &= 1.2, & \tilde{c}_X^{I'=3} &= 1.2, \end{aligned} \quad (4.26)$$

These charges satisfy (3.31) for the first two generations of quark and lepton multiplets. With this charge assignment, the observed quark and charged lepton masses and the absolute values of CKM mixings are realized, as shown in Table 4, with $\mathcal{O}(1)$ values of the holomorphic Yukawa couplings $\lambda_{ij}^{u,d,e}$ in the superpotential (4.10).

After fixing all the $U(1)_{I'}$ charges, the remaining parameters are the coefficients $\xi_{I'}^r$ in the effective gauge kinetic functions $f_{\text{eff}}^r(T)$ in (4.10). One of $\xi_{I'}^r$ for each r is determined by matching $f_{\text{eff}}^r(\langle T \rangle)$ with the observed values of the SM gauge couplings, *i.e.*, by the condition for the gauge coupling unification at M_{GUT} . The remaining $\xi_{I'}^r$ control the gaugino masses M_r at M_{GUT} . In the following analysis, all the gauge couplings, Yukawa couplings, the μ -term and soft SUSY breaking parameters in the visible sector are evaluated at the EW scale M_{EW} by using the 1-loop renormalization group (RG) equations of MSSM, where we neglect effects of all the Yukawa couplings except for the top Yukawa coupling.

In order to estimate the SUSY flavor violations, we rotate the soft scalar mass matrices $(m_{\tilde{f}_{L,R}}^2)_{ij} = \text{diag} \left(m_{\tilde{f}_{L,R1}}^2, m_{\tilde{f}_{L,R2}}^2, m_{\tilde{f}_{L,R3}}^2 \right)$ and the scalar trilinear coupling matrices $(\tilde{A}^f)_{ij} = (y^f)_{ij}(A^f)_{ij}$ into the super-CKM basis and describe them as

$$\hat{m}_{\tilde{f}_L}^2 = (V_L^f)^\dagger m_{\tilde{f}_L}^2 V_L^f, \quad \hat{m}_{\tilde{f}_R}^2 = (V_R^f)^\dagger m_{\tilde{f}_R}^2 V_R^f, \quad \hat{A}^f = (V_L^f)^\dagger \tilde{A}^f V_R^f, \quad (4.27)$$

	Predicted	Observed
$(m_u, m_c, m_t)/m_t$	$(1.4 \times 10^{-5}, 7.38 \times 10^{-3}, 1.0)$	$(1.5 \times 10^{-5}, 7.37 \times 10^{-3}, 1.0)$
$(m_d, m_s, m_b)/m_b$	$(1.2 \times 10^{-3}, 2.41 \times 10^{-2}, 1.0)$	$(1.2 \times 10^{-3}, 2.54 \times 10^{-2}, 1.0)$
$(m_e, m_\mu, m_\tau)/m_\tau$	$(2.871 \times 10^{-4}, 5.955 \times 10^{-2}, 1.0)$	$(2.871 \times 10^{-4}, 5.959 \times 10^{-2}, 1.0)$
$ V_{CKM} $	$\begin{pmatrix} 0.97324 & 0.2298 & 0.00337 \\ 0.2297 & 0.97235 & 0.042 \\ 0.00637 & 0.0417 & 0.999112 \end{pmatrix}$	$\begin{pmatrix} 0.97428 & 0.2253 & 0.00347 \\ 0.2252 & 0.97345 & 0.041 \\ 0.00862 & 0.0403 & 0.999152 \end{pmatrix}$

Table 4: The predicted quark and charged lepton masses as well as the absolute values of CKM mixings compared with the experimental data [29]. The flavor charges are chosen as shown in Eqs.(4.24) and (4.25).

where $f = u, d, e$ and $V_L^u = V_L^d \equiv V_L^q$. The unitary matrices $V_{L,R}^f$ are defined by

$$(V_L^f)^\dagger y_f V_R^f = \frac{1}{v_f} \text{diag} (m_{f1}, m_{f2}, m_{f3}), \quad (4.28)$$

where $v_f = (\sin \beta, \cos \beta, \cos \beta)v$ and $v \simeq 174$ GeV. Here we consider a case of $\tan \beta = 4$ as an example.¹⁷ Then we define mass insertion parameters as [30]

$$\begin{aligned} (\delta_{LL}^f)_{ij} &= \frac{(\hat{m}_{\hat{f}_L}^2)_{ij} + ((m_f)_i - \rho_{LL}^f) \delta_{ij}}{\sqrt{(\hat{m}_{\hat{f}_L}^2)_{ii}(\hat{m}_{\hat{f}_L}^2)_{jj}}}, & (\delta_{RR}^f)_{ij} &= \frac{(\hat{m}_{\hat{f}_R}^2)_{ij} + ((m_f)_i - \rho_{RR}^f) \delta_{ij}}{\sqrt{(\hat{m}_{\hat{f}_R}^2)_{ii}(\hat{m}_{\hat{f}_R}^2)_{jj}}}, \\ (\delta_{LR}^f)_{ij} &= \frac{v_f(\hat{A}^f)_{ij} - \mu_f(m_f)_i \delta_{ij}}{\sqrt{(\hat{m}_{\hat{f}_L}^2)_{ii}(\hat{m}_{\hat{f}_R}^2)_{jj}}} = (\delta_{RL}^f)_{ji}^*, \end{aligned} \quad (4.29)$$

where

$$\begin{aligned} \mu_f &= (\cot \beta, \tan \beta, \tan \beta) \mu, \\ \rho_{LL}^f &= \frac{\cos 2\beta}{6} (M_Z^2 - 4M_W^2, M_Z^2 + 2M_W^2, -3M_Z^2 + 6M_W^2), \\ \rho_{RR}^f &= \frac{\cos 2\beta}{3} \sin^2 \theta_W (-2M_Z^2, M_Z^2, 3M_Z^2), \\ M_Z &\simeq 91.2 \text{ GeV}, \quad M_W \simeq 80.1 \text{ GeV}, \quad \sin^2 \theta_W \simeq 0.23, \end{aligned} \quad (4.30)$$

and μ is determined by the minimization condition of the Higgs potential.

Furthermore we introduce a quantity

$$\Delta_\mu \equiv \frac{\mu}{2M_Z^2} \frac{\partial M_Z^2}{\partial \mu}, \quad (4.31)$$

¹⁷ In fact, $\tan \beta$ is not a free parameter in our original setup. One way to treat $\tan \beta$ as a free parameter is to introduce another μ -term, $\zeta_L \mathcal{H}_u \mathcal{H}_d$, on the $y = L$ boundary. Then the constant ζ_0 in (4.11) and (4.14) is replaced by $\zeta \equiv \zeta_0 + e^{-(c_{\mathcal{H}_u} + c_{\mathcal{H}_d}) \cdot T} \zeta_L$, and we can control the value of $\tan \beta$ by varying ζ_L keeping $\langle \zeta \rangle$ unchanged.

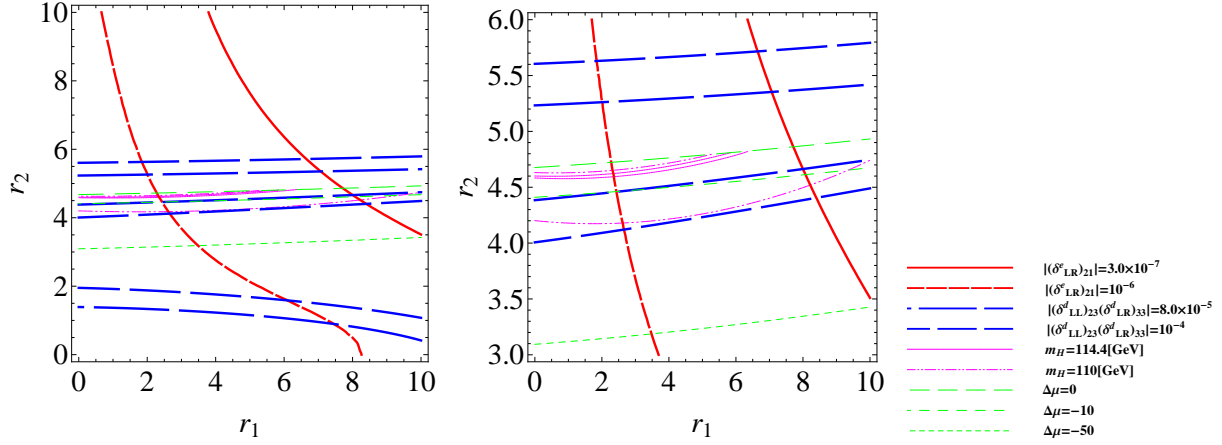


Figure 5: Contours of $|(\delta^e_{LR})_{21}|$, $|(\delta^d_{LL})_{23}(\delta^d_{LR})_{33}|$, $|\Delta_\mu|$ and m_{H_0} as functions of the gaugino mass ratios $r_1 = M_1/M_3$ and $r_2 = M_2/M_3$ at M_{GUT} . The region $3 \leq r_2 \leq 6$ is magnified in the right panel. The parameters are chosen as $M_{\text{SB}} = 100$ GeV, $\alpha_1 = 1$, $\alpha_2 = 1/2$, $\alpha_3 = 1/(4\pi^2)$, and $\tan\beta = 4$. The gluino mass M_3 is set to 343 GeV at M_{GUT} .

which describes the sensitivity of the Z-boson mass M_Z to the μ parameter [31].

In Figs. 5, 6 and 7, we plot contours of various quantities as functions of the gaugino mass ratios at M_{GUT} ,

$$r_1 \equiv M_1/M_3, \quad r_2 \equiv M_2/M_3. \quad (4.32)$$

Contours of $|(\delta^e_{LR})_{21}|$, $|(\delta^d_{LL})_{23}(\delta^d_{LR})_{33}|$, $|\Delta_\mu|$ and the lightest CP-even neutral Higgs mass m_{H_0} are shown in Fig. 5 for $M_{\text{SB}} = 100$ GeV, $\alpha_1 = 1$, $\alpha_2 = 1/2$ and $\alpha_3 = 1/(4\pi^2)$. Similar contours are plotted in Fig. 6 for $M_{\text{SB}} = 200$ GeV, $\alpha_1 = 1/2$, $\alpha_2 = 1/(8\pi^2)$ and $\alpha_3 = 1/(4\pi^2)$, and in Fig. 7 for $M_{\text{SB}} = 500$ GeV, $\alpha_1 = \alpha_3 = 1/(4\pi^2)$ and $\alpha_2 = 2/(4\pi^2)$. The gluino mass M_3 at M_{GUT} is 343 GeV, 383 GeV and 416 GeV in Figs. 5, 6 and 7, respectively. Ratios of the gaugino masses at M_{EW} to those at M_{GUT} are given by

$$\frac{M_1(M_{\text{EW}})}{M_1(M_{\text{GUT}})} = 0.4, \quad \frac{M_2(M_{\text{EW}})}{M_2(M_{\text{GUT}})} = 0.8, \quad \frac{M_3(M_{\text{EW}})}{M_3(M_{\text{GUT}})} = 2.9. \quad (4.33)$$

The gaugino masses at M_{EW} are read off from these relations.

The curves with fixed values of m_{H_0} represents the upper bound on the lightest CP-even Higgs mass in our model [32]. The region surrounded by the curve representing $|\Delta_\mu| = 10$ is free from the little hierarchy problem of MSSM. There is no (less than 10%) fine-tuning of μ in this region in order to realize the correct EW symmetry breaking.

As for the SUSY flavor violations, the most stringent experimental constraints on the mass insertion parameters in our model are typically expressed as $|(\delta^e_{LR})_{21}| \lesssim 10^{-7}$ and $|(\delta^d_{LL})_{23}(\delta^d_{LR})_{33}| \lesssim 10^{-4}$ which come from the upper bound on the branching ratios of $\mu \rightarrow e\gamma$ and $b \rightarrow s\gamma$, respectively [33]. Recall that contribution of the direct mediation by F^X has only weak flavor dependence for our charge assignment while that of the moduli mediation by $F^{T'}$ can disturb such flavor structures. Namely, the flavor dependence of the soft masses becomes weaker when $\alpha_{I'} \ll 1$. We emphasize that the allowed region from $\mu \rightarrow e\gamma$ is much wider in Fig. 6 with $\alpha_2 = 1/(8\pi^2)$, compared with the one in Fig. 5 with

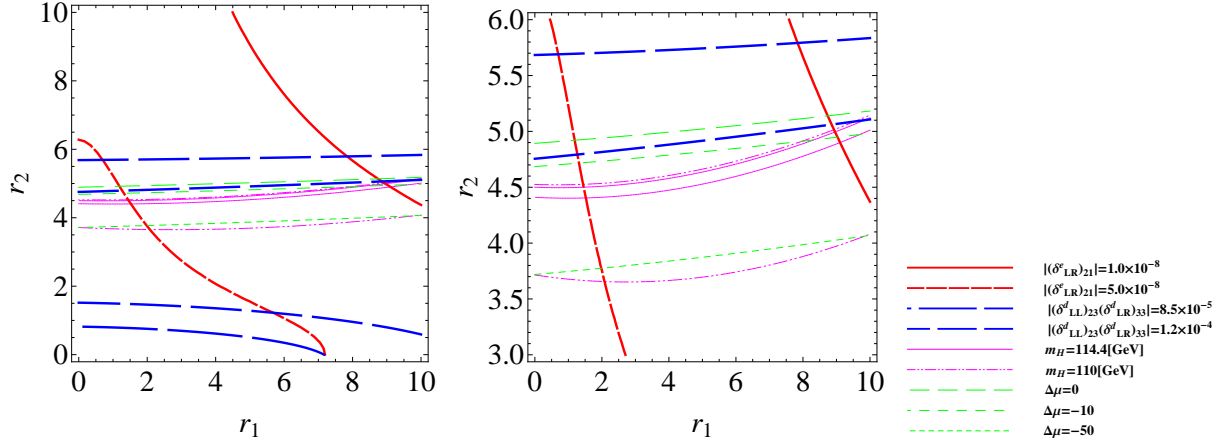


Figure 6: Contours of $|(\delta^e_{LR})_{21}|$, $|(\delta^d_{LL})_{23}(\delta^d_{LR})_{33}|$, $|\Delta_\mu|$ and m_{H_0} as functions of the gaugino mass ratios $r_1 = M_1/M_3$ and $r_2 = M_2/M_3$ at M_{GUT} . The region $3 \leq r_2 \leq 6$ is magnified in the right panel. The parameters are chosen as $M_{\text{SB}} = 200$ GeV, $\alpha_1 = 1/2$, $\alpha_2 = 1/(8\pi^2)$, $\alpha_3 = 1/(4\pi^2)$, and $\tan\beta = 4$. The gluino mass M_3 is set to 383 GeV at M_{GUT} .

$\alpha_2 = 1/2$. This is because the flavor structure of the first and second generations in the lepton sector are governed by the $U(1)_2$ charges, and only α_2 affects the flavor structure. Note that the contours of $|(\delta^d_{LL})_{23}(\delta^d_{LR})_{33}|$ in Fig. 7 is drawn differently compared with Figs. 5 and 6. This is because contributions of the direct mediation dominates over those of the moduli mediation since $\alpha_{I'} \ll 1$ in Fig. 7, and then the scaling of the soft terms are changed.

It is commonly said that models with the gravity-mediated SUSY breaking suffer from the SUSY flavor problem. However, we find that suitable charge assignments for $U(1)_{I'}$ do not cause the SUSY flavor problem while realize a viable flavor structure for quarks and leptons (without tachyonic squarks and sleptons). We should emphasize that this is due to the existence of multiple moduli, which induce additional contact terms $|Q_a|^2|X|^2$ in 4D effective Kähler potential. This is in sharp contrast to models with a single modulus discussed in many papers.

Finally we comment that the Higgs mass bound seems to be most stringent in Figs. 5, 6 and 7. This is due to the fact that the visible sector is assumed to be MSSM with $\tan\beta = 4$. It may become milder without affecting the flavor structure if we take a larger value of $\tan\beta$ ¹⁸ and/or extend the Higgs sector, such as the next to minimal SUSY SM. Since we are focusing on the flavor structure here, we would leave analyses on such extensions for a separate paper. Even for MSSM with $\tan\beta = 4$, we find a region where all the experimental constraints considered here are satisfied in Fig. 6.

¹⁸ For large values of $\tan\beta$, contributions of the bottom Yukawa coupling become important in the RG running, which we have neglected here.

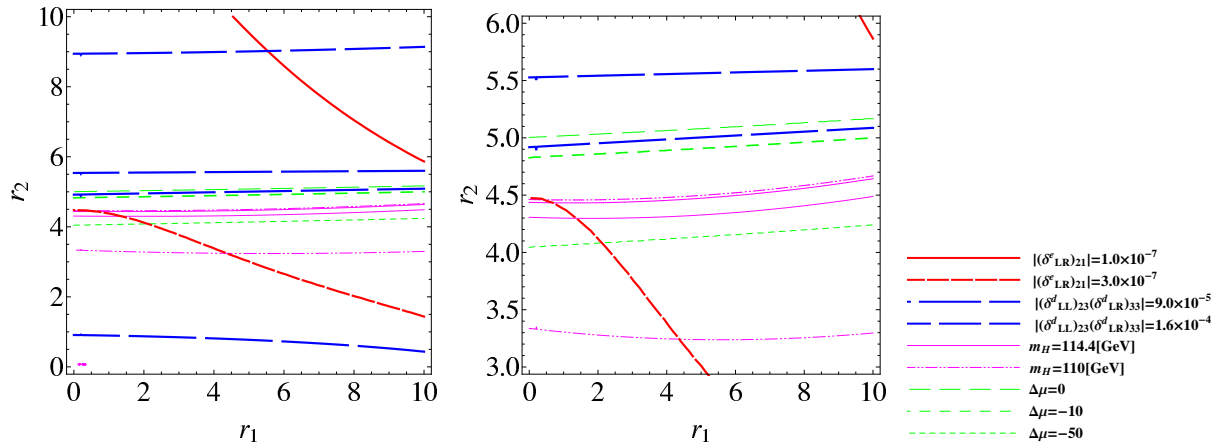


Figure 7: Contours of $|(\delta^e_{LR})_{21}|$, $|(\delta^d_{LL})_{23}(\delta^d_{LR})_{33}|$, $|\Delta_\mu|$ and m_{H_0} as functions of the gaugino mass ratios $r_1 = M_1/M_3$ and $r_2 = M_2/M_3$ at M_{GUT} . The region $3 \leq r_2 \leq 6$ is magnified in the right panel. The parameters are chosen as $M_{\text{SB}} = 500$ GeV, $\alpha_1 = 1/(4\pi^2)$, $\alpha_2 = 2/(4\pi^2)$, $\alpha_3 = 1/(4\pi^2)$, and $\tan \beta = 4$. The gluino mass M_3 is set to 416 GeV at M_{GUT} .

5 Summary

We have systematically studied the SUSY flavor structure of generic 5D SUGRA models, where all the hidden and visible sector fields are living in the whole 5D bulk spacetime, where $N = 2$ SUSY exists. In order to realize the observed quark and lepton masses and mixings, the visible sector fields are quasi-localized in the extra dimension by a suitable charge assignment for Z_2 -odd $U(1)_{I'}$ vector multiplets $V^{I'}$. This type of models have been considered in many papers, but most of them assume that there is just a single Z_2 -odd vector multiplet, *i.e.*, the graviphoton multiplet. However, it has been shown in Ref. [12] that induced squark and slepton masses become tachyonic in such a case. Besides, too large flavor violation generically occurs in the SUGRA models, *i.e.*, the SUSY flavor problem. In our previous work [13], we pointed out a new possibility of avoiding such problems by introducing an extra Z_2 -odd vector multiplet other than the graviphoton multiplet. In such a case, additional contributions to 4D effective Kähler potential K_{eff} appear after integrating out the Z_2 -odd vector multiplets, and they affect the flavor structure of the soft SUSY-breaking mass matrices.

In this paper, we have extended our previous work to more generic cases and specify conditions to solve the tachyonic sfermion problem (3.30) and the SUSY flavor problem (3.31). In fact, through a detailed phenomenological analysis, we have explicitly shown that the SUSY flavor problem can be avoided by introducing multiple vector multiplets $V^{I'}$ without encountering tachyonic sfermion problem mentioned above. Therefore we conclude that the SUSY flavor structure of gravity-mediated SUSY breaking scenario can be *controllable* [34] once it is concretely constructed, contrary to the general criticism that SUSY flavor violation is problematic for it.

The additional contributions to K_{eff} by integrating out the bulk SUGRA fields have been discussed in the context of the string theory in Ref. [35]. Because 5D SUGRA is the simplest set-up for the brane-world models, we can derive an explicit form of K_{eff} directly

from the higher-dimensional theory. This enables us to perform detailed analyses on it, as we have done in this paper. Our systematic analyses also owe to the existence of the off-shell description of SUGRA [4, 5]. It makes the derivation of the 4D effective theory transparent by utilizing the off-shell dimensional reduction [6].

The results obtained in this paper are quite generic when the hierarchical flavor structure originates from the wave function localization in the extra dimension. Most of the results in Sec. 4 do not much depend on the choice of the norm function (4.6) if we choose a suitable charge assignment for $V^{I'}$. Finally we emphasize that the multi moduli case discussed in this paper is naturally realized when we consider the low-energy effective theories of the string theory.

Acknowledgments

This work was supported in part by the Waseda University Grant for Special Research Projects No.2011B-177 (H. A.), and Grant-in-Aid for Young Scientists (B) No.22740187 from Japan Society for the Promotion of Science (Y. S.). H. A. and Y. S. thank the Yukawa Institute for Theoretical Physics at Kyoto University. Discussions during the YITP workshop "Summer Institute 2011" were useful to complete this work.

A Derivation of effective action

In this section, we provide a systematic derivation of the 4D effective action. We basically follow the off-shell dimensional reduction developed in Refs. [6, 13]. This procedure keeps the $N = 1$ off-shell structure.

First, we neglect the kinetic terms for Z_2 -odd $N = 1$ multiplets because they do not have zero-modes that are dynamical below the compactification scale. Then the Z_2 -odd multiplets play a role of Lagrange multipliers, and their equations of motion extract zero-modes from the Z_2 -even multiplets. This is the basic strategy.

A.1 Gauge kinetic functions and superpotential

The kinetic terms for $\Sigma^{I''}$ are included through the gauge-invariant quantities $\mathcal{V}^{I''}$ in the $d^4\theta$ -integral of (2.16). Thus, dropping the kinetic terms for $\Sigma^{I''}$ in a gauge-invariant way is equivalent to imposing the constraint,

$$\mathcal{V}^{I''} = 0. \quad (\text{A.1})$$

Before dropping the kinetic terms for the other Z_2 -odd multiplets, let us redefine the chiral multiplets $\Phi^{\hat{a}}$ as

$$\begin{aligned} \Phi^1 &= \phi^{\frac{3}{2}}\phi^c, & \Phi^2 &= \phi^{\frac{3}{2}}, \\ \Phi^{2a-1} &= \phi^{\frac{3}{2}}Q_a^c, & \Phi^{2a} &= \phi^{\frac{3}{2}}Q_a. \end{aligned} \quad (\text{A.2})$$

Then, ϕ has the Weyl weight 1 and plays a role of the chiral compensator multiplet, while Q_a has the Weyl weight zero, just like the matter chiral multiplets in 4D off-shell SUGRA.

Now we drop the kinetic terms for ϕ^c and Q^c , and obtain

$$\mathcal{L} = -2 \left[\int d^2\theta \left\{ \phi^{\frac{3}{2}} \phi^c (\partial_y + 3k \cdot \Sigma) \phi^{\frac{3}{2}} - \sum_a \phi^{\frac{3}{2}} Q_a^c (\partial_y + 2c_a \cdot \Sigma) \left(\phi^{\frac{3}{2}} Q_a \right) \right\} + \text{h.c.} \right] + \dots, \quad (\text{A.3})$$

where the ellipsis denotes terms independent of ϕ^c and Q^c . Thus their equations of motion are read off as

$$(\partial_y + 3k \cdot \Sigma) \phi^{\frac{3}{2}} = 0, \quad (\partial_y + 2c_a \cdot \Sigma) \left(\phi^{\frac{3}{2}} Q_a \right) = 0. \quad (\text{A.4})$$

By solving these, we find the y -dependence of ϕ and Q_a as

$$\begin{aligned} \phi(y) &= \exp \left\{ -2k \cdot \int_0^y dy' \Sigma(y') \right\} \phi(0), \\ Q_a(y) &= \exp \left\{ (3k - 2c_a) \cdot \int_0^y dy' \Sigma(y') \right\} Q_a(0), \end{aligned} \quad (\text{A.5})$$

where the 4D superspace coordinates x^μ and θ^α in the arguments are suppressed. Note that these y -dependent factors can be eliminated by 5D gauge transformations,

$$\begin{aligned} \tilde{\phi} &= e^{-2k \cdot \Lambda} \phi, \quad \tilde{Q}_a = e^{(3k - 2c_a) \cdot \Lambda} Q_a, \\ \tilde{V}^I &= V^I + \Lambda^I + \bar{\Lambda}^I, \quad \tilde{\Sigma}^I = \Sigma^I + \partial_y \Lambda^I = 0, \end{aligned} \quad (\text{A.6})$$

with the transformation parameters,

$$\Lambda^I(y) \equiv - \int_0^y dy' \Sigma^I(y'). \quad (\text{A.7})$$

Since $\Sigma^{I'}$ are Z_2 -even, the transformation parameters $\Lambda^{I'}$ are discontinuous at $y = L$. The discontinuities correspond to zero-modes for $\Sigma^{I'}$ as

$$\Lambda^{I'}|_{y=L-\epsilon} = -\frac{1}{2} T^{I'}, \quad \Lambda^{I'}|_{y=L} = 0, \quad (\text{A.8})$$

where

$$T^{I'} \equiv 2 \int_0^L dy \Sigma^{I'}(y). \quad (\text{A.9})$$

The zero-modes $T^{I'}$ are called the moduli multiplets in the following¹⁹. As a result, $\tilde{V}^{I'}$ also have gaps at $y = L$ as

$$\tilde{V}^{I'}|_{y=L-\epsilon} = -\text{Re } T^{I'}, \quad \tilde{V}^{I'}|_{y=L} = 0. \quad (\text{A.10})$$

After the above gauge transformation, the expressions of \mathcal{V}^I reduce to

$$\mathcal{V}^I = -\partial_y \tilde{V}^I. \quad (\text{A.11})$$

¹⁹ In the single modulus case, it corresponds to the radion multiplet T_{rad} .

Thus the constraint (A.1) indicates that the gauge-transformed Z_2 -even vector superfield $\tilde{V}^{I'}$ are independent of y , *i.e.*, 4D vector superfields.

Under the gauge transformation, \mathcal{L}_{vec} defined in (2.3) is invariant up to a total derivative for y [36],²⁰

$$\delta \mathcal{L}_{\text{vec}} = \int d^2\theta \partial_y \left(-\frac{C_{IJK}}{2} \Lambda^I \tilde{\mathcal{W}}^J \tilde{\mathcal{W}}^K \right) + \text{h.c.} \quad (\text{A.12})$$

This becomes surface terms after the y -integration, and thus

$$\begin{aligned} \int dy \delta \mathcal{L}_{\text{vec}} &= \int d^2\theta \left[-\frac{C_{I'J''K''}}{2} \Lambda^{I'} \tilde{\mathcal{W}}^{J''} \tilde{\mathcal{W}}^{K''} + \dots \right]_0^{L-\epsilon} + \text{h.c.} \\ &= \int d^2\theta \left\{ \frac{C_{I'J''K''}}{4} T^{I'} \tilde{\mathcal{W}}^{J''} \tilde{\mathcal{W}}^{K''} + \dots \right\} + \text{h.c.}, \end{aligned} \quad (\text{A.13})$$

where the ellipsis denotes terms involving a Z_2 -odd vector superfield $\tilde{V}^{I'}$. We have used (A.8) in the second equality.

Finally we drop the kinetic terms for $\tilde{V}^{I'}$, and perform the y -integration. By utilizing all the above results, we obtain

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \left[\int d^2\theta \frac{C_{I'J''K''}}{4} T^{I'} \tilde{\mathcal{W}}^{J''} \tilde{\mathcal{W}}^{K''} + \text{h.c.} \right] \\ &\quad - 3 \int d^4\theta |\tilde{\phi}|^2 \left\{ \int_0^{L-\epsilon} dy \hat{\mathcal{N}}^{1/3} (-\partial_y \tilde{V}) e^{2k \cdot \tilde{V}} \left(1 - \sum_{a=2}^{n_H+1} e^{(2c_a-3k) \cdot \tilde{V}} |\tilde{Q}_a|^2 \right)^{2/3} \right\} \\ &\quad + \left[\int d^2\theta \tilde{\phi}^3 \left\{ W^{(0)}(\tilde{Q}_a) + e^{-3k \cdot T} W^{(L)} \left(e^{(\frac{3}{2}k - c_a) \cdot T} \tilde{Q}_a \right) \right\} + \text{h.c.} \right], \end{aligned} \quad (\text{A.14})$$

where $\hat{\mathcal{N}}$ is a truncated norm function defined by

$$\hat{\mathcal{N}}(X) \equiv C_{I'J'K'} X^{I'} X^{J'} X^{K'}. \quad (\text{A.15})$$

Here we have used the fact that the gauge-transformed $\tilde{\phi}$ and \tilde{Q}_a are y -independent in the region $0 \leq y < L$ but has discontinuities at $y = L$ as

$$\tilde{\phi}|_{y \neq L} = e^{k \cdot T} \tilde{\phi}|_{y=L}, \quad \tilde{Q}_a|_{y \neq L} = e^{(c_a - \frac{3}{2}k) \cdot T} \tilde{Q}_a|_{y=L}. \quad (\text{A.16})$$

The chiral multiplets $\tilde{\phi}$ and \tilde{Q}_a in (A.14) are the ones evaluated at $y \neq L$.

From (A.14), we can read off the gauge kinetic functions and the superpotential in the effective theory as (3.2). We omit the tilde in the expression in (3.1) and (3.2).

Notice that the only y -dependent superfields in (A.14) are $\tilde{V}^{I'}$. Since we have dropped the kinetic terms for $\tilde{V}^{I'}$, we can integrate them out by their equations of motion. We will perform this procedure and calculate the effective Kähler potential in the next subsection.

²⁰ We have dropped total derivatives for x^μ and θ^α .

A.2 Kähler potential

Now we integrate $\tilde{V}^{I'}$ out in the expression (A.14), and derive the effective Kähler potential K_{eff} , or $\Omega_{\text{eff}} \equiv -3e^{-K_{\text{eff}}/3}$. It is written as

$$\Omega_{\text{eff}} = -3 \int_0^{L-\epsilon} dy \hat{\mathcal{N}}^{1/3} (-\partial_y \tilde{V}) e^{2k \cdot \tilde{V}} \left(1 - \sum_a e^{2d_a \cdot \tilde{V}} |\tilde{Q}_a|^2 \right)^{2/3}, \quad (\text{A.17})$$

where $d_{aI'} \equiv c_{aI'} - \frac{3}{2}k_{I'}$. We will omit the prime of the indices I', J', \dots in the following.

At first sight, it seems to be possible to integrate \tilde{V}^I out by finding their function forms in terms of y and performing the y -integral. However, this naive procedure fails because we cannot solve the equations of motion as functions of y . In fact, the equations of motion for \tilde{V}^I are rewritten as

$$\left\{ \partial_y \left(\frac{\hat{\mathcal{N}}_J}{\hat{\mathcal{N}}^{2/3}} \right) + 6k_J \hat{\mathcal{N}}^{1/3} - \frac{4 \sum_a d_{aJ} e^{2d_a \cdot \tilde{V}} |\tilde{Q}_a|^2}{1 - \sum_b e^{2d_b \cdot \tilde{V}} |\tilde{Q}_b|^2} \hat{\mathcal{N}}^{1/3} \right\} \mathcal{P}_I^J = 0, \quad (\text{A.18})$$

where the argument $(-\partial_y \tilde{V})$ of the norm function are suppressed, and the projection operator \mathcal{P}_I^J is defined in (2.13). The presence of \mathcal{P}_I^J indicates that the number of independent equations is less than that of \mathcal{V}^I . Thus we cannot solve \mathcal{V}^I as functions of y . This stems from the fact that 5D vector multiplets cannot be expanded into KK modes keeping $N = 1$ off-shell structure because component fields have different physical degrees of freedom within the multiplets, as mentioned in Sec. 2.4. Hence we need another method to integrate \tilde{V}^I out.

Let us define

$$v^I \equiv \frac{\partial_y \tilde{V}^I}{\partial_y U}, \quad U \equiv k \cdot \tilde{V}. \quad (\text{A.19})$$

Then $\mathcal{F}_J \equiv \hat{\mathcal{N}}_J / \hat{\mathcal{N}}^{2/3}$ is a function of v^I , and

$$\begin{aligned} \hat{\mathcal{N}}^{1/3} (-\partial_y \tilde{V}) &= \hat{\mathcal{N}}^{1/3} (v) \cdot (-\partial_y U), \\ \partial_y \mathcal{F}_J &= \partial_y v^K \mathcal{F}_{KJ} = -2 \partial_y v^K \hat{\mathcal{N}}^{1/3} (v) (a \cdot \mathcal{P})_{KJ} (v), \end{aligned} \quad (\text{A.20})$$

where a_{IJ} is defined in (2.6). Thus (A.18) is rewritten as

$$\left\{ \partial_y v^K a_{KJ} + \left(3k_J - \frac{2 \sum_a d_{aJ} e^{2d_a \cdot \tilde{V}} |\tilde{Q}_a|^2}{1 - \sum_b e^{2d_b \cdot \tilde{V}} |\tilde{Q}_b|^2} \right) \partial_y U \right\} \mathcal{P}_I^J = 0. \quad (\text{A.21})$$

Here and henceforth, the arguments of the norm function and its derivatives are understood as v^I unless specified. From (A.21) and the constraint $k_I (dv^I / dU) = 0$, we obtain

$$\begin{aligned} \frac{dv^I}{dU} &= a^{IJ} \left(-3\tilde{k}_J + \frac{2 \sum_a \tilde{d}_{aJ} e^{2d_a \cdot \tilde{V}} |\tilde{Q}_a|^2}{1 - \sum_b e^{2d_b \cdot \tilde{V}} |\tilde{Q}_b|^2} \right) \\ &= \mathcal{G}^I (v) + 2 \sum_a a^{IJ} \tilde{d}_{aJ} e^{2d_a \cdot \tilde{V}} |\tilde{Q}_a|^2 \\ &\quad + 2 \sum_{a,b} a^{IJ} \tilde{d}_{aJ} e^{2(d_a + d_b) \cdot \tilde{V}} |\tilde{Q}_a|^2 |\tilde{Q}_b|^2 + \mathcal{O}(|\tilde{Q}|^6), \end{aligned} \quad (\text{A.22})$$

where a^{IJ} is an inverse matrix of a_{IJ} , $\mathcal{G}^I(v) \equiv -3a^{IJ}\tilde{k}_J$, and

$$\begin{aligned}\tilde{k}_J &\equiv k_J - \frac{k_K a^{KL} k_L}{k_K a^{KL} \hat{\mathcal{N}}_L} \hat{\mathcal{N}}_J = (k \cdot \mathcal{P})_J - \frac{k \cdot \mathcal{P} a^{-1} \cdot k}{2\hat{\mathcal{N}}} \hat{\mathcal{N}}_J, \\ \tilde{d}_J &\equiv d_{aJ} - \frac{k_K a^{KL} d_{aL}}{k_K a^{KL} \hat{\mathcal{N}}_L} \hat{\mathcal{N}}_J = (d_a \cdot \mathcal{P})_J - \frac{k \cdot \mathcal{P} a^{-1} \cdot d_a}{2\hat{\mathcal{N}}} \hat{\mathcal{N}}_J.\end{aligned}\quad (\text{A.23})$$

Here we have used that

$$k_I a^{IJ} \hat{\mathcal{N}}_J = 2\hat{\mathcal{N}}, \quad (\text{A.24})$$

which follows from the relation $a^{IJ} \hat{\mathcal{N}}_J = 2\hat{\mathcal{N}} v^I$.

Since \tilde{V}^I can be regarded as a function of U through

$$\tilde{V}^I = \int_0^y dy' \partial_y \tilde{V}^I = \int_0^y dy' v^I \partial_y U = \int_0^U dU' v^I(U'), \quad (\text{A.25})$$

the y -integral in (A.17) can be converted into the integral for U . Thus, if we find explicit function forms of v^I in terms of U , we can integrate \tilde{V}^I out by using the boundary conditions,

$$U|_{y=0} = 0, \quad U|_{y=L-\epsilon} = -k \cdot \text{Re } T. \quad (\text{A.26})$$

This is actually possible (at least in principle) because the number of independent v^I is equal to that of the equations of motion due to the constraint $k_I v^I = 1$.

We expand v^I as

$$v^I = \bar{v}^I(U) + \sum_a g_a^I(U) |\tilde{Q}_a|^2 + \sum_{a,b} g_{ab}^I(U) |\tilde{Q}_a|^2 |\tilde{Q}_b|^2 + \mathcal{O}(|\tilde{Q}|^6). \quad (\text{A.27})$$

From (A.22), the functions $\bar{v}^I(U)$, $g_a^I(U)$ and $g_{ab}^I(U)$ satisfy

$$\begin{aligned}\frac{d\bar{v}^I}{dU} &= \mathcal{G}^I(\bar{v}), \\ \frac{dg_a^I}{dU} &= \mathcal{G}_J^I(\bar{v}) g_a^J(U) + 2 \left(a^{IJ} \cdot \tilde{d}_{aJ} \right) \Big|_{v=\bar{v}} e^{2 \int_0^U dU' d_a \cdot \bar{v}}, \\ \frac{dg_{ab}^I}{dU} &= \mathcal{G}_J^I(\bar{v}) g_{ab}^J + \frac{1}{2} \mathcal{G}_{JK}^I(\bar{v}) g_a^J g_b^K + 2e^{2 \int_0^U dU' (d_a + d_b) \cdot \bar{v}} \left(a^{IJ} \cdot \tilde{d}_{aJ} \right) \Big|_{v=\bar{v}} \\ &\quad + 2e^{2 \int_0^U dU' d_a \cdot \bar{v}} \left\{ \partial_K \left(a^{IJ} \cdot \tilde{d}_{aJ} \right) \Big|_{v=\bar{v}} g_b^K(U) + 2 \left(a^{IJ} \cdot \tilde{d}_{aJ} \right) \Big|_{v=\bar{v}} \int_0^U dU' d_a \cdot g_b(U') \right\},\end{aligned}\quad (\text{A.28})$$

where $\mathcal{G}_J^I(v) \equiv \partial \mathcal{G}^I / \partial v^J$ and $\mathcal{G}_{JK}^I(v) \equiv \partial^2 \mathcal{G}^I / \partial v^J \partial v^K$. From (A.25) and (A.26), it follows that

$$-\text{Re } T^I = \int_0^{-k \cdot \text{Re } T} dU v^I(u), \quad (\text{A.29})$$

which leads to

$$\int_0^{-k \cdot \text{Re } T} dU \bar{v}^I(U) = -\text{Re } T^I, \quad \int_0^{-k \cdot \text{Re } T} dU g_a^I(U) = \int_0^{-k \cdot \text{Re } T} dU g_{ab}^I(U) = 0. \quad (\text{A.30})$$

Substituting (A.27) into (A.17), we obtain

$$\begin{aligned}
\Omega_{\text{eff}} &= 3 \int_0^{-k \cdot \text{Re } T} dU \hat{\mathcal{N}}^{1/3}(v) e^{2U} \left(1 - \sum_a e^{2 \int_0^U dU' d_a \cdot v} |\tilde{Q}_a|^2 \right)^{2/3} \\
&= 3 \int_0^{-k \cdot \text{Re } T} dU e^{2U} \left[\hat{\mathcal{N}}^{1/3}(\bar{v}) + \sum_a \left\{ \frac{\mathcal{F}_I(\bar{v})}{3} g_a^I - \frac{2\hat{\mathcal{N}}^{1/3}(\bar{v})}{3} e^{2d_a \cdot \bar{V}} \right\} |\tilde{Q}_a|^2 \right. \\
&\quad + \sum_{a,b} \left\{ \frac{\mathcal{F}_I(\bar{v})}{3} g_{ab}^I + \frac{\mathcal{F}_{IJ}(\bar{v})}{6} g_a^I g_b^I - \frac{2\mathcal{F}_I(\bar{v})}{9} g_a^I e^{2d_b \cdot \bar{V}} \right. \\
&\quad \left. \left. - \hat{\mathcal{N}}^{1/3}(\bar{v}) \left(\frac{4}{3} e^{2d_a \cdot \bar{V}} \int_0^U dU' d_a \cdot g_b + \frac{e^{2(d_a+d_b) \cdot \bar{V}}}{9} \right) \right\} |\tilde{Q}_a|^2 |\tilde{Q}_b|^2 \right] \\
&\quad + \mathcal{O}(|\tilde{Q}|^6),
\end{aligned} \tag{A.31}$$

where

$$\bar{V}^I(U) \equiv \int_0^U dU' \bar{v}^I(U'). \tag{A.32}$$

Now we will find the function forms of $\bar{v}^I(U)$, $g_a^I(U)$ and $g_{ab}^I(U)$. Since it is difficult to find general solutions to the differential equations in (A.28), we focus on a simple case where the condition,

$$k_I \mathcal{P}^I_J(\bar{v}) = 0, \tag{A.33}$$

is satisfied. In this case, $\mathcal{G}^I(\bar{v}) = 0$ and \bar{v}^I reduces to a constant for U . By using (A.30), it is determined as

$$\bar{v}^I = \frac{\text{Re } T^I}{k \cdot \text{Re } T}. \tag{A.34}$$

Thus the condition (A.33) means that

$$k_I \mathcal{P}^I_J(\text{Re } T) = 0. \tag{A.35}$$

Substituting (A.34) into the second equation in (A.28), we obtain

$$g_a^I(U) = \left\{ e^{2d_a \cdot \bar{v}U} - Y(d_a \cdot T) Y \left(\frac{k \cdot T}{2} \mathcal{G} \right)^{-1} e^{\mathcal{G}U} \right\}_J^I C_a^J, \tag{A.36}$$

where

$$C_a^I \equiv \left[2(-\mathcal{G} + 2d_a \cdot \bar{v})^{-1} \cdot a^{-1} \cdot \tilde{d}_a \right]^I, \quad Y(z) \equiv \frac{1 - e^{-2\text{Re } z}}{2\text{Re } z}, \tag{A.37}$$

and \mathcal{G} is a matrix whose (I, J) -component is $\mathcal{G}_J^I(\bar{v})$. Under the constraint (A.33), we find that $k \cdot a^{-1} \cdot \tilde{d}_a = 0$ and $\mathcal{G}^I_J(\bar{v}) = -2(\delta^I_J - k_J \bar{v}^I)$, which indicates that

$$\left(\mathcal{G} a^{-1} \tilde{d}_a \right)^I = -2 \left(a^{-1} \tilde{d}_a \right)^I. \tag{A.38}$$

Thus (A.36) is rewritten as

$$g_a^I(U) = \frac{1}{1 + d_a \cdot \bar{v}} \left\{ e^{2d_a \cdot \bar{v}U} - \frac{Y(d_a \cdot T)}{Y(-k \cdot T)} e^{-2U} \right\} \left(a^{-1} \cdot \tilde{d}_a \right)^I. \tag{A.39}$$

Substituting (A.34) and (A.39) into the third equation in (A.28), we obtain

$$g_{ab}^I(U) = \left(e^{\mathcal{G}^U} \right)_J^I \int_{U_0}^U dU' \left(e^{-\mathcal{G}^{U'}} \right)_K^J C_{ab}^K(U'), \quad (\text{A.40})$$

where U_0 is a constant, which is determined by (A.30), and

$$\begin{aligned} C_{ab}^I(U) \equiv & \frac{1}{2} \mathcal{G}_{JK}^I(\bar{v}) g_a^J g_b^K + 2e^{2(d_a+d_b) \cdot \bar{v} U} \left(a^{IJ} \cdot \tilde{d}_{aJ} \right) \Big|_{v=\bar{v}} \\ & + 2e^{2d_a \cdot \bar{v} U} \left\{ \partial_K \left(a^{IJ} \cdot \tilde{d}_{aJ} \right) \Big|_{v=\bar{v}} g_b^K(U) + 2 \left(a^{IJ} \cdot \tilde{d}_{aJ} \right) \Big|_{v=\bar{v}} \int_0^U dU' d_a \cdot g_b(U') \right\}. \end{aligned} \quad (\text{A.41})$$

Under the constraint (A.33), we can show that

$$\hat{\mathcal{N}}_I(\bar{v}) C_{ab}^I(U) = 0, \quad (\text{A.42})$$

which leads to

$$\mathcal{F}_I(\bar{v}) g_{ab}^I = e^{-2U} \int_{U_0}^U dU' e^{2U'} \mathcal{F}_I(\bar{v}) C_{ab}^I(U') = 0. \quad (\text{A.43})$$

Substituting (A.34) and (A.39) into (A.31) and using (A.43), we obtain the final result.

$$\Omega_{\text{eff}} = \Omega^{(0)} + \sum_a \Omega_a^{(2)} |\tilde{Q}_a|^2 + \sum_{a,b} \Omega_{a,b}^{(4)} |\tilde{Q}_a|^2 |\tilde{Q}_b|^2 + \mathcal{O}(|\tilde{Q}|^6), \quad (\text{A.44})$$

where

$$\begin{aligned} \Omega^{(0)} &= -3\hat{\mathcal{N}}^{1/3}(\text{Re } T) Y(k \cdot T), \\ \Omega_a^{(2)} &= 2\hat{\mathcal{N}}^{1/3}(\text{Re } T) Y((k + d_a) \cdot T), \\ \Omega_{a,b}^{(4)} &= \hat{\mathcal{N}}^{1/3}(\text{Re } T) \left[-\frac{(d_a \cdot \mathcal{P} a^{-1} \cdot d_b) \left\{ Y((k + d_a + d_b) \cdot T) - \frac{Y(d_a \cdot T) Y(d_b \cdot T)}{Y(-k \cdot T)} \right\}}{\{(k + d_a) \cdot \text{Re } T\} \{(k + d_b) \cdot \text{Re } T\}} \right. \\ &\quad \left. + \frac{Y((k + d_a + d_b) \cdot T)}{3} \right]. \end{aligned} \quad (\text{A.45})$$

We have used various relations stemming from the fact that $\hat{\mathcal{N}}$ is a cubic polynomial. In (3.2) in the text, we omit the tilde of \tilde{Q}_a .

B Comment on integration out of $V^{I'}$

Here we provide a comment on the effect of integrating out the Z_2 -odd vector multiplets $V^{I'}$. As mentioned in Sec. 3.5, this procedure induces the quartic terms for Q_a in the effective Kähler potential. Let us denote the components of Q_a as $Q_a = q_a + \theta \chi_{Q_a} + \theta^2 F_{Q_a}$. Then we have four-fermion interactions coming from the quartic terms in Ω_{eff} ,

$$\mathcal{L}_{4\text{-fermi}} = |\langle \phi \rangle|^2 \frac{\hat{\mathcal{N}}^{1/3}(\text{Re } \langle T \rangle)}{8} \sum_{a,b} \tilde{\Omega}_{a,b}^{(4)} |\chi_{Q_a} \chi_{Q_b}|^2. \quad (\text{B.1})$$

To simplify the explanation, let us consider a case that $(k + d_a) \cdot \text{Re} \langle T \rangle < 0$ and $d_b \cdot \text{Re} \langle T \rangle > 0$. Then, $Y((k + d_a + d_b) \cdot \langle T \rangle) \ll \frac{Y(d_a \cdot \langle T \rangle)Y(d_b \cdot \langle T \rangle)}{Y(-k \cdot \langle T \rangle)}$ and the above four-fermion terms are proportional to $d_a \cdot \mathcal{P}a^{-1} \cdot d_b$. However, since all the vector components in $V^{I'}$ including the graviphoton are physical, the contributions of the diagrams in Fig. 1 to $\mathcal{L}_{4\text{-fermi}}$ are expected to be proportional to $d_a \cdot a^{-1} \cdot d_b$, rather than $d_a \cdot \mathcal{P}a^{-1} \cdot d_b$. Note that the coefficient of the first term in (3.4) is rewritten as

$$\frac{d_a \cdot \mathcal{P}a^{-1} \cdot d_b}{\{(k + d_a) \cdot \text{Re} T\} \{(k + d_b) \cdot \text{Re} T\}} = \frac{d_a \cdot a^{-1} \cdot d_b}{\{(k + d_a) \cdot \text{Re} T\} \{(k + d_b) \cdot \text{Re} T\}} - \frac{2}{3}, \quad (\text{B.2})$$

where we have used (3.5). The second term in the left-hand side corresponds to the lack of the graviphoton contribution. In fact, it is restored when the promotion (2.5) is taken into account. For example, each derivative ∂_μ is promoted to the covariant derivative \mathcal{D}_μ that includes the $U(1)_A$ gauge field A_μ . ($U(1)_A$ is the R-symmetry of the $N = 1$ superconformal algebra.) This gauge field does not have a kinetic term in the action, and is an auxiliary field. By integrating out the auxiliary fields in the Weyl multiplet including A_μ , we obtain an additional contribution to (B.1), and cancel the second term in (B.2). Notice that this contribution does not originate from the $|Q_a|^2|Q_b|^2$ terms in Ω_{eff} , but is purely SUGRA interaction. Hence it is not accompanied with a term $|q_a|^2|F_{Q_b}|^2$, which contributes to the soft SUSY-breaking scalar masses.

As a result, the dominant part of the four-fermion interaction is proportional to $d_a \cdot a^{-1} \cdot d_X$, while that of the soft scalar mass is to $d_a \cdot \mathcal{P}a^{-1} \cdot d_X$, as shown in Sec. 3.5.

C Explicit expressions in the illustrative model

Here we collect explicit expressions in the illustrative model discussed in Sec. 4.

From the definition $K_{\text{eff}} = -3 \ln(-\Omega_{\text{eff}}/3)$ and (3.2), we obtain the Kähler metric as

$$\begin{aligned} K_{I' \bar{J}'} &\equiv \partial_{T^{I'}} \partial_{\bar{T}^{J'}} K_{\text{eff}} = \frac{1}{2} a_{I' J'} - \frac{3}{4} k_{I'} k_{J'} \mathcal{Y}(k \cdot \text{Re} T), \\ K_{Q_a \bar{Q}_b} &\equiv \partial_{Q_a} \partial_{\bar{Q}_b} K_{\text{eff}} = \frac{2Y((k + d_a) \cdot T)}{Y(k \cdot T)} \delta_{ab}, \\ K_{X \bar{X}} &\equiv \partial_X \partial_{\bar{X}} K_{\text{eff}} = \frac{2Y((k + d_X) \cdot T)}{Y(k \cdot T)}, \end{aligned} \quad (\text{C.1})$$

where $\mathcal{Y}(x)$ is defined from $Y(x)$ (x : real) in (3.3) as

$$\mathcal{Y}(x) \equiv \frac{Y \partial_x^2 Y - (\partial_x Y)^2}{Y^2}(x) = \frac{1 + e^{4x} - 2e^{2x}(1 + 2x^2)}{(1 - e^{2x})^2 x^2}. \quad (\text{C.2})$$

The function $\mathcal{Y}(x)$ is an even function and monotonically decreasing function of $|x|$. For example, $\mathcal{Y}(0) = 1/3$ and $\mathcal{Y}(\pm \kappa L) \simeq 0.032$ when $\kappa L = 5.6$.

In Sec. 4, we have assumed that only $F^{T^{I'}}$ ($I' = 1, 2, 3$) and F^X have nonvanishing VEVs. Thus, the soft scalar mass normalized by M_{SB} defined in (4.1) is calculated as

$$\frac{m_a^2}{M_{\text{SB}}^2} \simeq \frac{1}{K_{X \bar{X}}} \left\{ -\frac{\tilde{\Omega}_{a,X}^{(4)}(\text{Re} \langle T \rangle)}{Y(\tilde{c}_a)} + \frac{F^{T^{I'}} \bar{F}^{T^{J'}}}{|F^X|^2} \left(\frac{a_{I' J'}}{6} - \frac{1}{4} (k + d_a)_{I'} (k + d_a)_{J'} \mathcal{Y}(\tilde{c}_a) \right) \right\}, \quad (\text{C.3})$$

where $\tilde{c}_a \equiv \sum_{I'} \tilde{c}_a^{I'}$.

For the (truncated) norm function chosen as (4.6), we can obtain explicit forms of various functions of $\mathcal{X}^{I'}$, where $I' = 1, 2, 3$ is the index of the Z_2 -odd vector multiplets. The coefficient matrix of the vector boson kinetic term $a_{I'J'}$ defined in (2.6) is calculated as

$$a_{I'J'}(\mathcal{X}) = \begin{pmatrix} \frac{1}{2(\mathcal{X}^1)^2} & & \\ & \frac{1}{2(\mathcal{X}^2)^2} & \\ & & \frac{1}{2(\mathcal{X}^3)^2} \end{pmatrix}, \quad (\text{C.4})$$

and the projection operator \mathcal{P}^I_J defined in (2.13) is

$$\mathcal{P}^I_J(\mathcal{X}) = \frac{1}{3} \begin{pmatrix} 2 & -\frac{\mathcal{X}^1}{\mathcal{X}^2} & -\frac{\mathcal{X}^1}{\mathcal{X}^3} \\ -\frac{\mathcal{X}^2}{\mathcal{X}^1} & 2 & -\frac{\mathcal{X}^2}{\mathcal{X}^3} \\ -\frac{\mathcal{X}^3}{\mathcal{X}^1} & -\frac{\mathcal{X}^3}{\mathcal{X}^2} & 2 \end{pmatrix}. \quad (\text{C.5})$$

Thus, the symmetric matrix $\mathcal{P}a^{-1}$ appearing in the expression of $\tilde{\Omega}_{a,b}^{(4)}$ in (3.4) is obtained as

$$\{\mathcal{P}a^{-1}\}(\mathcal{X}) = \frac{2}{3} \begin{pmatrix} 2(\mathcal{X}^1)^2 & -\mathcal{X}^1\mathcal{X}^2 & -\mathcal{X}^1\mathcal{X}^3 \\ -\mathcal{X}^1\mathcal{X}^2 & 2(\mathcal{X}^2)^2 & -\mathcal{X}^2\mathcal{X}^3 \\ -\mathcal{X}^1\mathcal{X}^3 & -\mathcal{X}^2\mathcal{X}^3 & 2(\mathcal{X}^3)^2 \end{pmatrix}. \quad (\text{C.6})$$

References

- [1] N. Arkani-hamed and M. Schmaltz, *Phys. Rev.* **D61** (2000) 033005 [[hep-ph/9903417](#)]; D.E. Kaplan and T.M.P. Tait, *JHEP* **0006** (2000) 020 [[hep-ph/0004200](#)].
- [2] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83** (1999) 3370 [[hep-ph/9905221](#)].
- [3] G. Servant, T. M. P. Tait, *Nucl. Phys.* **B650** (2003) 391-419. [[hep-ph/0206071](#)].
- [4] T. Kugo and K. Ohashi, *Prog. Theor. Phys.* **105** (2001) 323 [[hep-ph/0010288](#)]; T. Fujita and K. Ohashi, *Prog. Theor. Phys.* **106** (2001) 221 [[hep-th/0104130](#)]; T. Fujita, T. Kugo and K. Ohashi, *Prog. Theor. Phys.* **106** (2001) 671 [[hep-th/0106051](#)].
- [5] T. Kugo and K. Ohashi, *Prog. Theor. Phys.* **108** (2002) 203 [[hep-th/0203276](#)].
- [6] H. Abe and Y. Sakamura, *Nucl. Phys.* **B796** (2008) 224 [[arXiv:0709.3791](#)].
- [7] F. Paccetti Correia, M. G. Schmidt and Z. Tavartkiladze, *Nucl. Phys.* **B709** (2005) 141 [[hep-th/0408138](#)].
- [8] H. Abe and Y. Sakamura, *JHEP* **0410** (2004) 013 [[hep-th/0408224](#)].
- [9] H. Abe, Y. Sakamura, *Nucl. Phys.* **B796** (2008) 224-245. [[arXiv:0709.3791](#) [[hep-th](#)]].
- [10] R. Altendorfer, J. Bagger, D. Nemeschansky, *Phys. Rev.* **D63** (2001) 125025. [[hep-th/0003117](#)]; A. Falkowski, Z. Lalak, S. Pokorski, *Phys. Lett.* **B491** (2000) 172-182. [[hep-th/0004093](#)]; T. Gherghetta, A. Pomarol, *Nucl. Phys.* **B586** (2000) 141-162. [[hep-ph/0003129](#)].

- [11] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, *Phys. Rev.* **D59** (1999) 086001 [[hep-th/9803235](#)]; *Nucl. Phys.* **B552** (1999) 246 [[hep-th/9806051](#)].
- [12] H. Abe, T. Higaki, T. Kobayashi, Y. Omura, *JHEP* **0804** (2008) 072. [[arXiv:0801.0998](#) [[hep-th](#)]].
- [13] H. Abe, Y. Sakamura, *Phys. Rev.* **D79**, 045005 (2009). [[arXiv:0807.3725](#) [[hep-th](#)]].
- [14] N. Arkani-Hamed, T. Gregoire, J. G. Wacker, *JHEP* **0203**, 055 (2002). [[hep-th/0101233](#)].
- [15] F. P. Correia, M. G. Schmidt and Z. Tavartkiladze, *Nucl. Phys.* **B751** (2006) 222 [[hep-th/0602173](#)].
- [16] M. Kaku, P.K. Townsend and P. Van Nieuwenhuizen, *Phys. Rev. Lett.* **39** (1977) 1109; *Phys. Lett.* **B69** (1977) 304; *Phys. Rev.* **D17** (1978) 3179; T. Kugo and S. Uehara, *Nucl. Phys.* **B226** (1983) 49; *Prog. Theor. Phys.* **73** (1985) 235.
- [17] Y. Sakamura, [[arXiv:1107.4247](#) [[hep-th](#)]].
- [18] D. Marti, A. Pomarol, *Phys. Rev.* **D64**, 105025 (2001). [[hep-th/0106256](#)].
- [19] E. Bergshoeff, R. Kallosh, A. Van Proeyen, *JHEP* **0010**, 033 (2000). [[hep-th/0007044](#)].
- [20] J. Wess and J. Bagger, *Princeton, USA: Univ. Pr. (1992) 259 p*
- [21] M. A. Luty, R. Sundrum, *Phys. Rev.* **D64**, 065012 (2001). [[hep-th/0012158](#)].
- [22] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski, S. Pokorski, *JHEP* **0411**, 076 (2004). [[hep-th/0411066](#)].
- [23] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski, *Nucl. Phys.* **B718**, 113-133 (2005). [[hep-th/0503216](#)].
- [24] H. Abe, T. Higaki, T. Kobayashi, Y. Omura, *Phys. Rev.* **D75**, 025019 (2007). [[hep-th/0611024](#)]; H. Abe, T. Higaki, T. Kobayashi, *Phys. Rev.* **D76**, 105003 (2007). [[arXiv:0707.2671](#) [[hep-th](#)]].
- [25] E. Dudas, C. Papineau, S. Pokorski, *JHEP* **0702**, 028 (2007). [[hep-th/0610297](#)].
- [26] A. E. Nelson, N. Seiberg, *Nucl. Phys.* **B416**, 46-62 (1994). [[hep-ph/9309299](#)].
- [27] V. S. Kaplunovsky and J. Louis, *Phys. Lett. B* **306** (1993) 269 [[arXiv:hep-th/9303040](#)].
- [28] H. Davoudiasl, J. L. Hewett, T. G. Rizzo, *Phys. Lett.* **B473** (2000) 43-49. [[hep-ph/9911262](#)].
- [29] K. Nakamura *et al.* [Particle Data Group Collaboration], *J. Phys. G* **G37**, 075021 (2010).

- [30] M. Misiak, S. Pokorski, J. Rosiek, Adv. Ser. Direct. High Energy Phys. **15**, 795-828 (1998). [hep-ph/9703442].
- [31] R. Barbieri, G. F. Giudice, Nucl. Phys. **B306**, 63 (1988).
- [32] M. S. Carena, M. Quiros, C. E. M. Wagner, Nucl. Phys. **B461**, 407-436 (1996). [hep-ph/9508343]; M. S. Carena, H. E. Haber, S. Heinemeyer, W. Hollik, C. E. M. Wagner, G. Weiglein, Nucl. Phys. **B580**, 29-57 (2000). [hep-ph/0001002].
- [33] H. Abe, K. Choi, K. -S. Jeong, K. -i. Okumura, JHEP **0409** (2004) 015. [hep-ph/0407005].
- [34] J. P. Conlon, JHEP **0803**, 025 (2008). [arXiv:0710.0873 [hep-th]].
- [35] A. Anisimov, M. Dine, M. Graesser, S. D. Thomas, Phys. Rev. **D65**, 105011 (2002). [hep-th/0111235].
- [36] E. Dudas, T. Gherghetta and S. Groot Nibbelink, Phys. Rev. D **70** (2004) 086012 [arXiv:hep-th/0404094].